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MECHANISMS IN THERMAL
MECHANICAL FORMING OF PLATES

by

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(1975)

Submitted to the Departments of Ocean Engineering and
Mechanical Engineering in partial fulfillment of the
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Abstract

The angular deflection produced by flame line heating
as plate dimensions are varied, specifically in the direc-
tion of heating, is examined. The behavior is found to be
similar to that observed when the line of heating approaches
a plate edge and the bending, in both cases, can be obtained
from the same parametric curve. The parameter values defin-
ing these curves are related to process variables: the ther-
mal power and speed of travel of the heat source. Addition-
ally, the in plate variation in bending is characterize.
    The application of these findings to the formulation of
a bending prediction model that incorporates plate dimen-
sions as well as location of the heating line is presented.
The extension of this model to predict bending at specific
locations within the plate is discussed, and recommendations
for further research are made.

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CHAPTER 1
INTRODUCTION

Industrial applications of metal plate forming processes are numerous. They range from forming the complex compound shapes of automobile body components to the various shapes required in the construction of ships and other marine structures.

The thin plate, high volume applications of the automobile industry are well suited to mechanical forming using press and die techniques. However, the thick plate low volume applications encountered in the shipbuilding and other large scale construction industries require other methods. The shaping of large, thick metal plate can be achieved by mechanical forming, thermo-mechanical forming, or some combination of the two. Mechanical forming of such large plate is limited to roller or break press methods, which are capable of producing curvature in one direction only, precluding the forming of complex shapes, such as in shell plating for a ship's bow or stern.

Thermo-mechanical methods, primarily that of line heating, have been used extensively for such shipbuilding applications for many years. In this technique, a heat source (usually an oxy-acetylene flame), is passed over the plate

surface to produce thermal stresses in the plate. These stresses in turn, cause yielding of the metal resulting in distortion of the plate. When implemented by highly skilled workers, who through years of practical experience have developed the art of determining how and where to apply the flame to produce desired shapes in a plate, the results are impressive. Compound shapes can be formed more rapidly and economically than by other methods, with less investment in capital equipment. However, the decreasing number of such highly skilled craftsmen in the work force, coupled with the continuing requirements of the shipbuilding industry, has emphasized a need for the development of new thermo-mechanical forming techniques that can be readily automated.

Toward this goal, the use of a laser heat source, with its increased stability and high power density, was proposed by Prof. Masubuchi. This resulted in the currently ongoing three year study, "*Laser Forming of Steel Plates for Ship Construction*," sponsored by the U. S. Navy and Todd Pacific Shipyards Corporation, under his direction. The first phase of this study included an investigation of primary and secondary factors influencing plate bending, the results of which are contained in the Phase I Report.[1] Although the effects of plate thickness and location of the heat source

were examined, the effect of varying the plate dimensions was not.

The investigation of the effects associated with such changes in plate dimensions, particularly in the direction of movement of the heat source, is the basis of this thesis.

CHAPTER 2
THE THERMO-MECHANICAL FORMING PROCESS

2.1 General Description

In the thermo-mechanical forming process, the controlled application of heat is used to produce local thermal expansion, resulting in transient thermal stresses that exceed the yield strength of the material. This yielding produces local regions of plastic (non-elastic) strain which result in deformation and movement of the material as it returns to ambient temperature. The actual process is complex, as it involves the temperature dependent properties of the metal in the non-uniform temperature field produced by the heat source. Examples of this temperature dependence for a representative mild steel are illustrated in Figures 2.1 through 2.3 [4]. The stress-strain relationships, determined by three of these properties (yield strength, Young's modulus, and strain hardening), therefore exhibit corresponding changes with temperature. Any formal analysis of the mechanism must be able to address the formidable problem of accounting for these changes in properties and relationships with temperature at each point within the material. An excellent treatment is given in

"Theory of Thermal Stresses" by B. A. Boley and J. H. Weiner [7]; particularly Chapters 14 and 16, dealing with the formulation and analysis of the problem, respectively. However, for the current purpose, a less formal discussion similar to that given by McCarthy,[2] is presented in the following paragraphs.

Fundamental to the forming process is the principal that plastic strain will occur only in the presence of some restraint that limits the thermal expansion of the material. As an illustration, consider the uniform heating of a totally unrestrained steel plate to just below its austentizing temperature, A_1, followed by uniform cooling to ambient temperature. Although the temperature exceeds that which would produce plastic yielding (flow) in a restrained plate, no deformation occurs. The uniform temperature field and the absence of any restraint allows the plate to expand and contract by the same amount during the thermal cycle. However, the presence of an external restraint or a sufficiently non-uniform temperature field will result in deformation.

The amount of plastic strain, produced whenever the thermal strain exceeds the yield strain of the material and defined as the difference of the two strains, represents the amount of deformation available at a given temperature. The

thermal strain produced is essentially the same for the various structural steels since the coefficient of thermal expansion exhibits slight change for these various steels. However, the yield strain, which is the quotient of the yield strength and Young's modulus of the material, varies significantly between the different steels. Thus, an upper limit to the amount of deformation that can be achieved by thermo-mechanical forming is established as a function of the yield strength of the material. It should be noted that due to the decrease of yield strength with increasing temperature, thermo-mechanical forming can produce bending with considerably lower and more uniform residual stress distributions, as compared to similar bending with direct mechanical forming methods as rolling or break press.

The most commonly practiced method of thermo-mechanical forming, as previously mentioned, is that of line heating in which no external restraint is applied. In this method, the heat source (usually an oxy-acetylene flame or laser) travels in a straight line along one axis of the plate. The restraint producing the plastic yielding is that of the cooler plate material surrounding that directly under the heat source. Thus, the temperature gradient in the plate has a significant effect on the amount of final

plate deformation produced.   The mechanism can be qualita-
tively described as follows:

As the plate material under the heat source is heated
beyond the point where plastic flow is initiated, this
region of the plate will expand more rapidly than the cooler
surrounding material, and will enter a state of compressive
loading. The difference in loading through the thickness of
the plate produces a moment that results in negative
(concave down) deformation of the plate. As the heat source
moves away, this region of the plate cools and the compres-
sive plastic deformation that occurred in the upper portion
of the plate produces a state of tension. The plate then
rotates deforming positively (concave up) as it seeks to
achieve equilibrium under this residual tensile stress.
Since the resultant force and resultant moment produced by
any residual stress must vanish, this implies that the
residual tensile stress must be balanced by a compressive
stress acting in the same section of the plate.

This description has served as the basis for various
analytical models of the thermo-mechanical bending in
plates. One such, the Heat Deformation model, developed by
Iwasaki, et al [5], is used in the analysis of the results
obtained during the current investigation, and will be dis-
cussed in more detail later in this chapter. Although the

above description of the thermo-mechanical process iden-
tified several major factors affecting the bending, there
are other factors that contribute to the behavior. A sum-
mary of these factors is presented in the following section.

2.2 Factors Affecting Bending

The factors known to influence the bending process can
be grouped according to their relative contribution to the
overall behavior, as follows.

Primary Factors:

1. Yield Strength-- The higher the yield strength,
implies that more thermal stress is required to produce
yielding, resulting in less deformation or bending for
a given heat input.

2. Heat Input-- The amount of deformation depends on
both the maximum temperature obtained at a point during
the heating and the temperature gradient in the vicin-
ity of that point, particularly in the thickness direc-
tion. The heat input to the plate surface, usually
expressed in terms of KJoules/inch, is primarily a
function of the power produced by the heat source
(KJoules /sec.) and the speed at which it moves over
the plate. Another consideration of more consequence

in laser applications, is the coupling of the heat
source to the plate. It should be noted that a higher
heat input does not necessarily imply greater bending.
If the power is too high or the speed too low, more
material is heated to higher temperatures, and less
restraint is available to produce plastic strain.
Similarly, if the power is too low or the speed too
high, the maximum temperature is decreased and less
plastic strain produced. A parameter that has proved
useful in relating the bending to the heat input is the
power divided by the square root of the speed, $P/\sqrt{v}$.

3. Plate Thickness-- The dependence of the bending on
the temperature gradient in the thickness direction
would indicate that for each plate thickness their
should exist some particular heat input that would
optimize the bending. However, as the plate thickness
increased, this heat input would also have to increase
until one of two practical limits are reached: 1) the
heat input would result in surface melting or 2) the
plate would become so thick that the volume of material
would prevent any motion, independent of the heat
input.

Secondary factors:

1. Edge Restraint-- The plastic flow of the heated material is dependent on the restraint afforded by the surrounding cooler material. This can be augmented by the application of additional external restraint, such as clamping an edge of the plate. The closer the heat source is applied to the edge restraint, the more pronounced the bending.

2. Heating Location-- Relating to the discussion of the previous paragraph is the reduced bending obtained as the heat source is applied closer to a free edge. Actually, this observed "edge effect" is the consequence of two factors: the reduced material available to act as a heat sink both in the thickness direction as well as along the surface of the plate (resulting in a reduced temperature gradient), and the reduced mechanical restraint.

3. Applied Cooling-- Improvements in bending efficiency should be obtained if the temperature gradient in the thickness direction is increased. This can be achieved by forced cooling of the back side (side opposite the heat source) of the plate.

4. Geometric Constraint-- The bending produced by the heat source is affected by the plate geometry. If the

plate is already curved along an axis perpendicular to
the line of heating, the moment of inertia would be
greater than if the plate were flat. This effect
influences the bending obtained when multiple passes at
right angles to each other are used to produce a com-
pound shape.

5.   Pre-strain Condition-- If the plate has existing
strain due to residual or imposed mechanical stresses
prior to heating, its behavior will be modified. The
presence of compressive strain will result in plastic
flow occurring at lower temperatures (i.e. less thermal
strain required for yielding). Conversely, the pres-
ence of tensile strain will raise the required thermal
strain, raising the temperature required to initiate
plastic flow. This is the primary cause for the
reduced bending observed in successive passes of a heat
source over the same line.

It is one of these secondary factors, namely that of
heating location, that is central to the current investiga-
tion. The variations in bending behavior with plate length
can be considered to be a direct consequence of this effect.
The influence of these "edge effects" in large plates has
been studied for the situation where the line of heating

is parallel to the edge in question.[1] The effect on bend-
ing of varying the distance between a parallel edge and the
line of heating is depicted in **Figure 2.4.** This behavior
should be symmetrical with respect to the plate center with
maximum bending occurring along the center line parallel to
the edges considered. In this case, it is clear that
decreasing edge to center line distance, as in a series of
progressively narrower plates, will result in an accompa-
nying decrease in bending. However, for the situation where
the distance between the plate edges perpendicular to the
line of heating is varied, the behavior could exhibit some
difference from that previously discussed. Additionally,
the bending produced at different locations along the line
of heating could also differ, particularly near the edges.
These possible differences being a consequence of the dif-
ferent orientation of the edges. An attempt to examine and
if possible, to characterize this behavior motivated this
investigation.

2.3 Empirical vs. Theoretical

The study of the behavior of plates subjected to tran-
sient thermal loading has produced several theoretical and
numerical models that can predict the ultimate residual
stress and strain distributions and resulting deformation.

The increasing capability and availability of computers has facilitated advances in the area of theoretical stress analysis. This has permitted the development of models, more capable of simulating the complex behavior of thermo-mechanical deformation. However, depending on the numerical methods used and approach taken, these models can be subject to several limitations. Some of these are practical i.e. computer time and cost required; others are analytic, i.e. the lack of an accurate thermo-elastic-plastic model that can simultaneously couple the thermal and mechanical effects. This last limitation results in the current practice of dividing the analysis into two parts: the thermal history (usually from experimental results) is established, and then served as input for the stress-strain analysis. A detailed discussion of this type of analysis, emphasizing the applications to the study of weld induced distortion, is found in Professor Masubuchi's *Analysis of Welded Structures.*[3] It should be noted that in many practical applications, complete theoretical models are not required. The ultimate goal of the current research project, directed by Professor Masubuchi, i.e. the development of automated processes for the thermo-mechanical forming of steel plate, especially the high strength steels used in Naval applications is an example. Computer prediction models based on

experimentally determined empirical data can be used to con-
trol the automated process. Consequently, it appears that
the most effective approach remains one of combined experi-
mental work with theoretical stress analysis.

Part of the current effort is to determine parameters
that will permit improved predictions when dealing with
plates of varying dimensions.

2.4 Specific Models

In the course of this investigation, two models were
considered for analysis of the experimental results and
merit additional discussion. The following paragraphs out-
line the Heat Deformation Model presented by Iwasaki, et al
[5], and the Elastic-Plastic Model presented by Iwamura and
Rybicki.[4]

Heat Deformation Model

The basic approach is that of considering the thermal
transient to occur in a series of discrete steps in both
space and time. This is accomplished by viewing the plate
section at three time frames. The initial time frame is
that of an unrestrained plate section of unit thickness at
ambient temperature. The second time frame considers the
plate section with the heat source directly above it and the

top surface at maximum temperature. At this point, the plate section is treated as if it consists of four regions: a top region of thickness n at a temperature T_h, a bottom region of thickness $1-n$ at a temperature T_r, and two side regions of unit thickness at ambient temperature that surround the heated regions. The values of n, T_h, and T_r are calculated using a model of the temperature field, and the plastic strain produced in the top region determined by the difference in the thermal strains experienced by the top and bottom regions less the yield strain of the material at T_h. The third time frame is that of the plate once again at ambient temperature, but now the compressive plastic strain formed during heating is producing tensile stress in the top region which is balanced by a compressive stress in the bottom region, resulting in bending. Application of equilibrium conditions and stress-strain relationships then provide an expression for the bending moment in terms of: the plastic strain produced, Young's modulus at ambient temperature, and the determined thickness of the T_h region, n. The predicted bending is then determined using the moment of inertia of the plate section and an effective width, w, also determined from the temperature field.

The procedure for determining n, T_h, T_r and w is fully discussed in the reference. But, it can be described as

follows: n is that distance from the top of the plate in
which yielding is expected to occur at maximum temperature;
T_h is the temperature at $n/2$; T_r is the bottom temperature
when the top surface is at maximum temperature; $w/2$ is that
distance from the center of the section where the tempera-
tures at n and the plate bottom are such that yielding no
longer occurs.

This model lends itself for very rapid real-time pre-
dictions of the bending performed on micro computers if pro-
vided with temperature inputs.

Iwamura and Rybicki's Model

This two dimensional model considers the temperature
dependent properties and includes elastic-plastic relations.
Although incremental loading is used, the assumption of the
stress paths for loading and unloading made by the authors
results in a simplified set of stress-strain relations. The
basic approach is to consider, for each load increment, that
the total strain in the plate depends on the two dimensional
variables, but in a linear manner with respect to each.
The mechanical strain is taken as the difference between the
total and thermal strain at each point. The total strain is
estimated by applying equilibrium conditions for the stress
through the plate thickness, where this stress is expressed
in terms of the total and thermal strains. This provides

the computational algorithm with the iteration required to solve the resulting system of non-linear equations.

The displacement and deflection is then calculated for each load increment at each point in the grid representing the plate. The accumulated effects at the end of the thermal cycle are then used to calculate the residual strain and stress distributions predicted for the plate along with the distortion and deflection.

The predicted behavior is in very good agreement with the results of experiments performed by the authors using a very narrow, but thick metal plate.

An attempt to use this method to develop a computer model to serve as an analytic tool during the current investigation was undertaken. However, problems in convergence of the iterative procedure could not be overcome in time to support this objective.
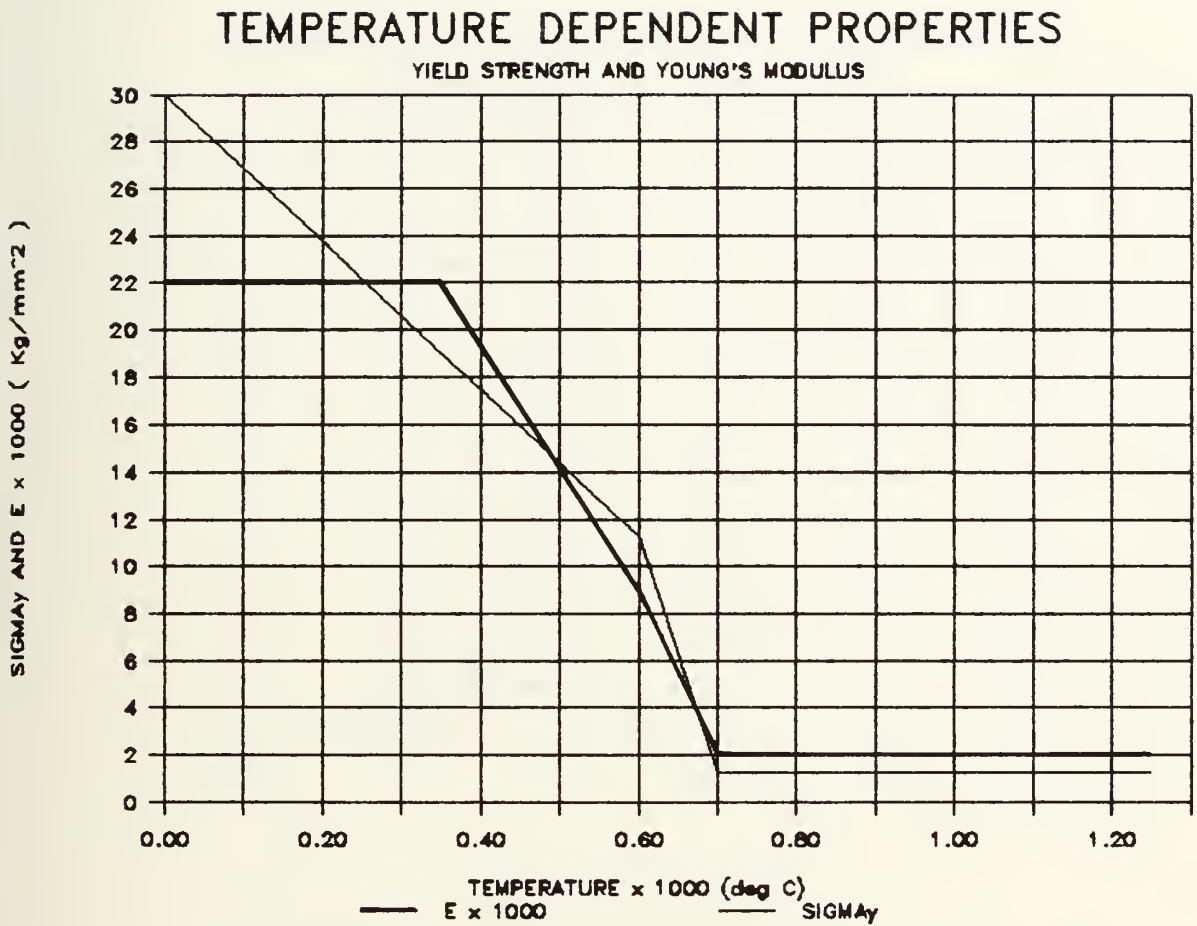
FIGURE 2.1   TEMPERATURE DEPENDENT PROPERTIES:

YIELD STRENGTH AND YOUNG'S MODULUS
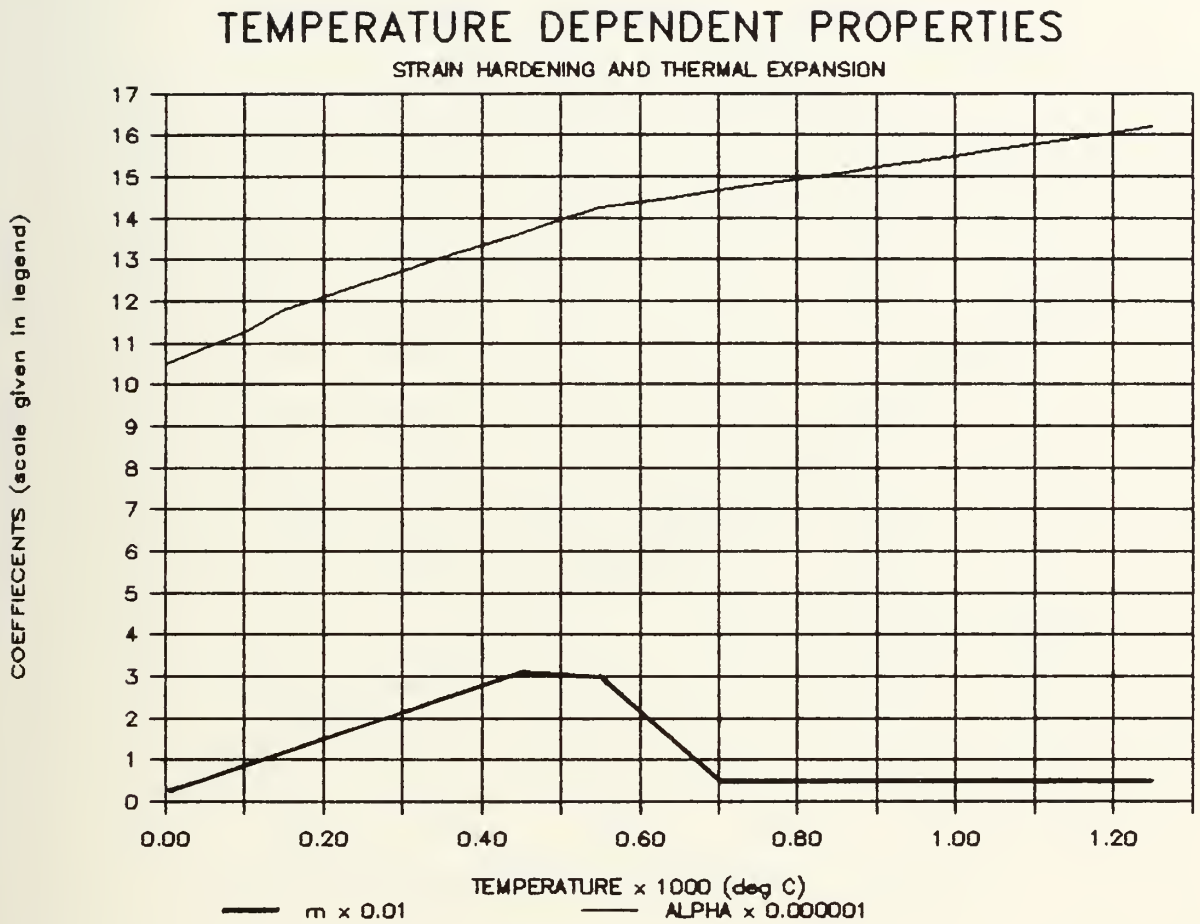
FIGURE 2.2 TEMPERATURE DEPENDENT PROPERTIES:
COEFFICIENTS OF STRAIN HARDENING AND THERMAL EXPANSION

TEMPERATURE DEPENDENT PROPERTIES

THERMAL STRAIN AND YIELD STRAIN
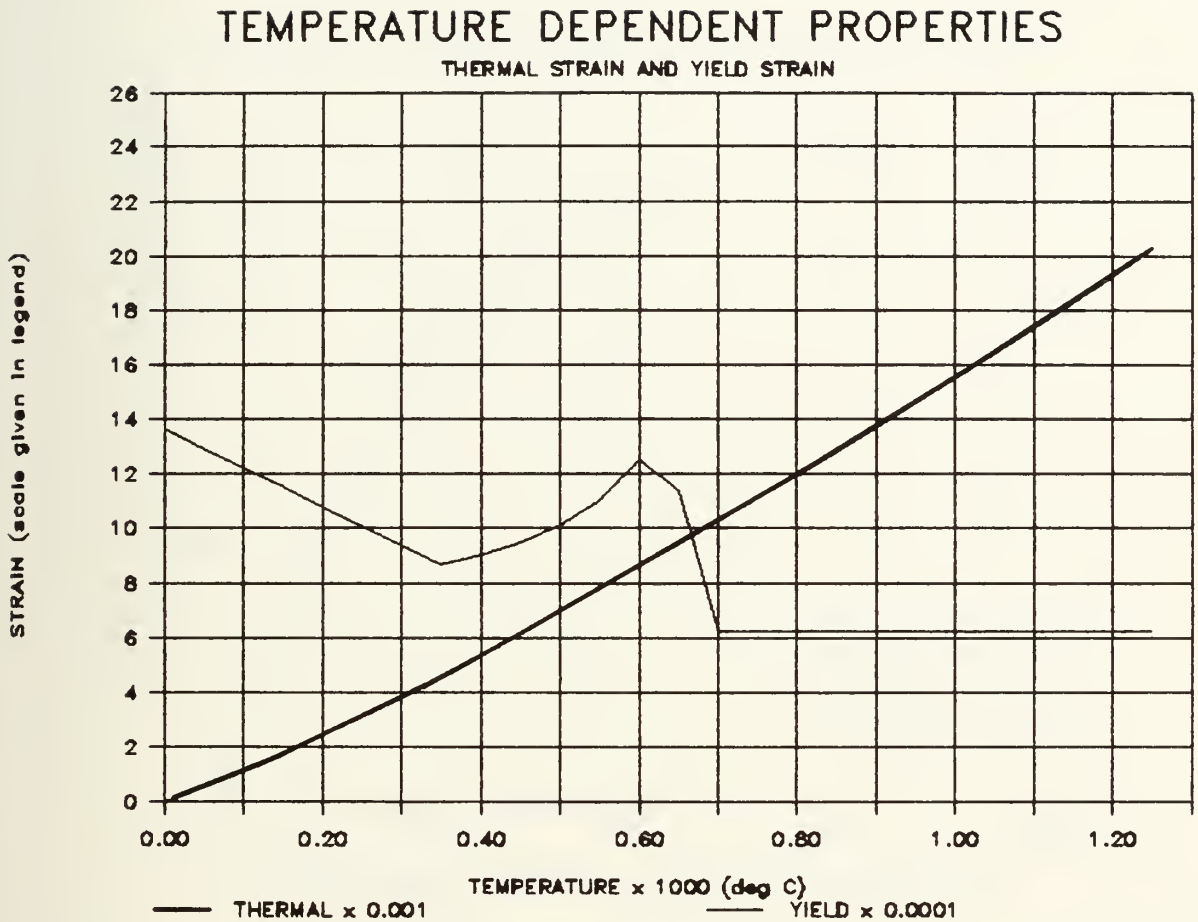
FIGURE 2.3   TEMPERATURE DEPENDENT PROPERTIES:

YIELD STRAIN AND THERMAL STRAIN
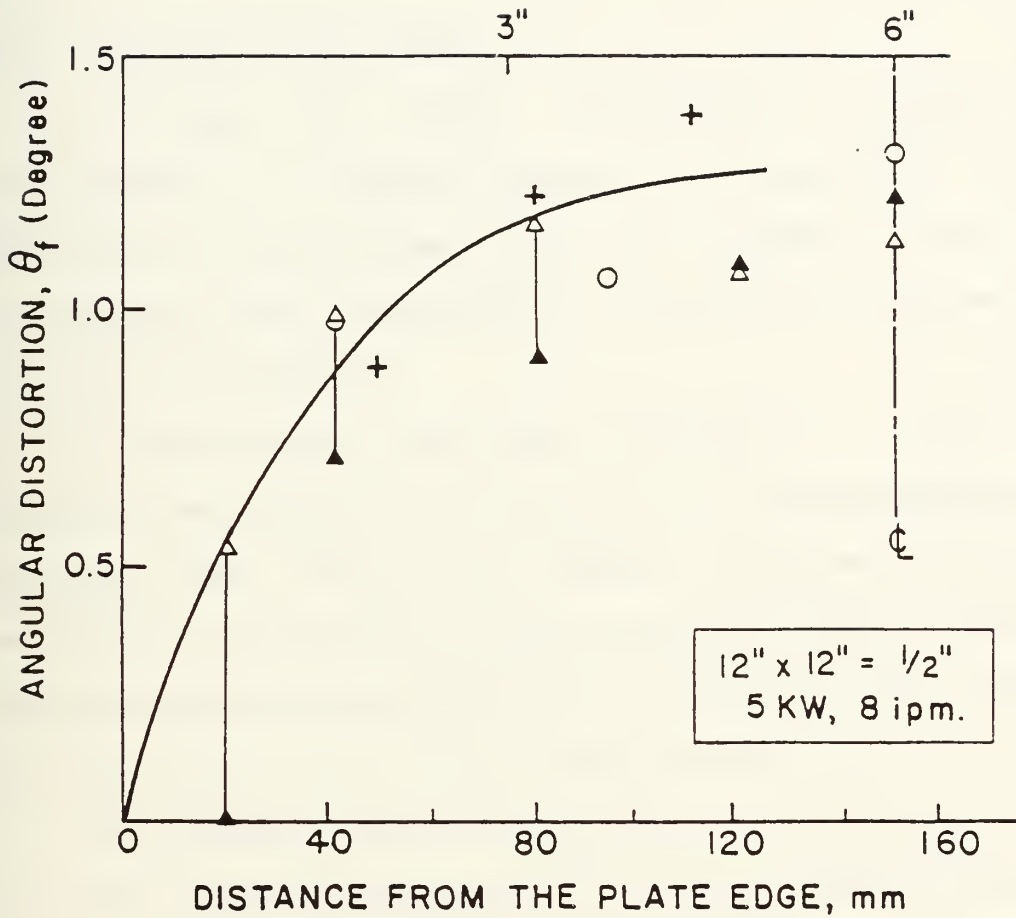
FIGURE 2.4 EFFECT OF LOCATION OF HEAT SOURCE:
DISTANCE FROM PLATE EDGE vs. θ_t

CHAPTER 3
EXPERIMENTAL PROCEDURES

3.1 General Approach

To study the overall dependence of plate bending (deflection) on length, plates of varying length but of the same width and thickness were line heated. The actual experiments were conducted in 3 parts as outlined in the following paragraphs.

A preliminary experiment, consisting of a series of 4 plates (designated 0-1 through 0-4), was conducted using a constant heat input. The results obtained indicated that subsequent experiments should include larger plates, and that as was expected, deflective behavior at the plate edges differed from that at the plate interior.

The largest plate of the series (12"x18"x3/8") was then cut into strips perpendicular to the direction of heating. The out-of-plane distortion after each cut, in the strips and the remaining plate, were recorded to provide data pertaining to the in plate deflective behavior noted.

Two follow-on experiments, each consisting of a series of 6 plates (designated 1-1 through 1-6 and 2-1 through 2-6), were then performed. One of these experiments was conducted at the same heat input as the preliminary

experiment; the other at a reduced heat input (approximately 1/2 that of the preliminary experiment). The heat input, expressed in terms of $P/V\bar{v}$, was estimated from temperature measurements obtained for each experiment.

3.2 Materials and Equipment

The metal plate used in each of the experiments was Hot-rolled ASTM A-36 Low Carbon Steel plate. This material was not subjected to any special treatment. No attempts to remove any existing residual stresses from the rolling process were made. The surface preparation consisted only of cleaning and did not include any machining, other than grinding of the edges to remove metal burrs from the cutting of the plates to the required size by the supplier.

The dimensions of the plates used for each experiment are given in the Tables 3.1 and 3.2.

Commercially available oxy-acetylene torches were used for all of the experiments. The larger torch (an AIRCO type 800 with a #7 tip) produced average temperatures of 1000 degrees C and 506 degrees C at the top and bottom surfaces respectively. The other torch (a PUROX type W-200 with a #6 tip) produced average temperatures of 508 degrees C and 221 degrees C at the top and bottom surfaces respectively.

The temperature measurements were made with an unshielded 1/8" diameter K-type Chromal-Alumel thermocouple mounted at the end of a ceramic tube for support. The thermocouple was connected to an electronic ice point reference device, whose voltage output (in millivolts) was read on a digital voltmeter. This voltage data was then converted to the corresponding temperatures. The choice of the bead size was dictated by the need for a relatively small thermocouple time constant.

The mechanical dial indicator used for distortion measurements had a range of 3 inches with an accuracy of 0.0005 inch (i.e. direct reading to the nearest 0.001 inch).

3.3 Experimental Setups

In all the experiments, the plates were simply supported along lines parallel to the line of heating, and at a distance of one fourth the plate width (4.5 inches) from each edge. The torch was mounted to a variable speed electro-mechanical carriage, set at a speed of 7.5 in/min +/- 0.5 in/min, to move the flame over the plates at constant speed and torch tip height. This arrangement is illustrated in Figure 3.1.

To determine initial (pre-heating) and final (post-heating) plate distortion, a flat (granite) table was used

to support the plates, and the mechanical dial indicator affixed to a flat block, was used to measure the out of plane distortion of the plates.

The cutting of the one plate from the preliminary experiment was done on a band saw at slow speed to minimize the introduction of additional distortion due to local heating in the vicinity of the cutting line.

Temperature readings were taken at four locations on a representative plate for each of the two heat input levels used. These four locations were the top and bottom plate surfaces both at near edge (0.5 inches from the edge), and at the plate interior (5 inches from the edge). These measurements were obtained using the bead type thermocouple in contact with the plate surface. The top surface measurements were taken at a point 0.5 to 0.75 inches behind the moving torch with the thermocouple bead shielded from the deflected hot gases from the torch flame by a ceramic plate, as illustrated in **Figure 3.2**. However, with the larger torch, the increased volume of deflected gases made it difficult to obtain accurate readings, even with the ceramic shield. The bottom surface measurements were taken immediately after the tip passed the indicated location.

3.4   Procedures

Each experiment was carried out in three steps or procedures which are detailed in the following paragraphs.

Initial Measurement Procedure.   This consisted of surface cleaning of each plate to be used in the experiment, followed by the marking of a reference grid on the top surface.   Then initial out of plane distortion of each plate over its surface was measured using the flat table and mechanical dial indicator and the raw data recorded.

Line Heating Procedure.   In preparation for the actual line heating, the plates were placed on the support structure.   Alignment of the plates and leveling of the support structure to ensure consistency of torch tip position and height over all the plates, was then performed. Finally, torch travel speed was set and measured by operating the carriage.   Then the actual line heating of the plates was performed.   The average torch speed was again measured during the heating.   The plates were then allowed to cool, in air, to room temperature.

Final Measurement Procedure.   The out of plane distortion of the cooled plates was then determined in the same manner as described for the initial measurements and the raw data recorded.

The raw data, in the form of the relative out of plane distortion, was then converted to angular deflection along each grid line and an average angular deflection for each plate calculated. To accomplish this, a linear regression of the raw distortion readings versus distance from the line of heating along each grid line was performed. This resulted in two straight lines, each perpendicular to and extending from the line of heating to the corresponding edge. The angular deflection along each grid line was then calculated using the slopes of these two lines. This was done for the initial (pre-heating) and final (post-heating) distortion readings at each grid line. The actual heat induced bending of the plates along each grid line, expressed as angular deflection, was then taken as the difference of the final and initial angular deflections for that grid line.

For the preliminary experiment, the additional procedure of cutting of the larger plate (0-4, 12"x18"x3/8") into 1.5 inch strips about each grid line perpendicular to the line of heating was performed. After each cut, the relative out-of-plane distortion at points along the grid lines on the plate as well as the cut strip were measured. This raw data was then converted to angular deflections, as outlined in the previous paragraph.

Preliminary Experiment (4 plates)

PLATE #	DIMENSIONS
0-1	3"X18"X3/8"
0-2	6"X18"X3/8"
0-3	9"X18"X3/8"
0-4	12"X18"X3/8"

Table 3.1   Plate Dimensions

Follow-On Experiments (6 plates)

PLATE #	DIMENSIONS
1-1, 2-1	3"X18"X3/8"
1-2, 2-2	6"X18"X3/8"
1-3, 2-3	9"X18"X3/8"
1-4, 2-4	12"X18"X3/8"
1-5, 2-5*	15"X18"X3/8"
1-6, 2-6*	18"X18"X3/8"

* THESE FOUR WERE STAMPED OUT (PRESS CUT) FROM A LARGER PLATE, INSTEAD OF CUT FROM PLATES HOT-ROLLED TO THEIR RESPECTIVE LENGTHS, AS THE SMALLER PLATES IN THE SERIES.

Table 3.2   Plate Dimensions
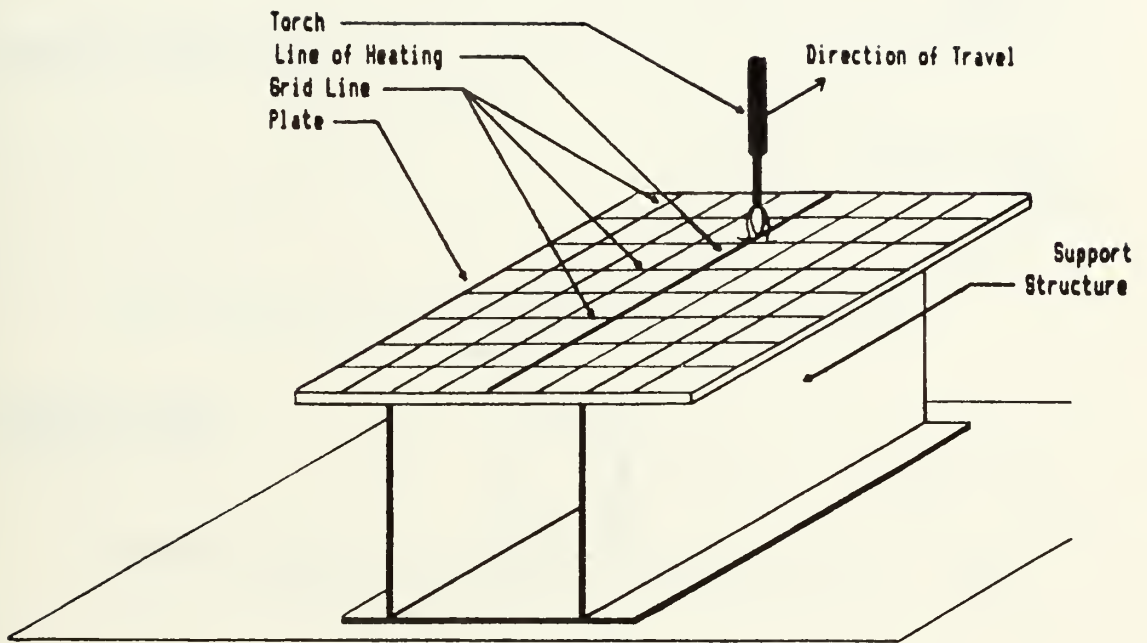
FIGURE 3.1   EXPERIMENTAL SETUP FOR LINE HEATING
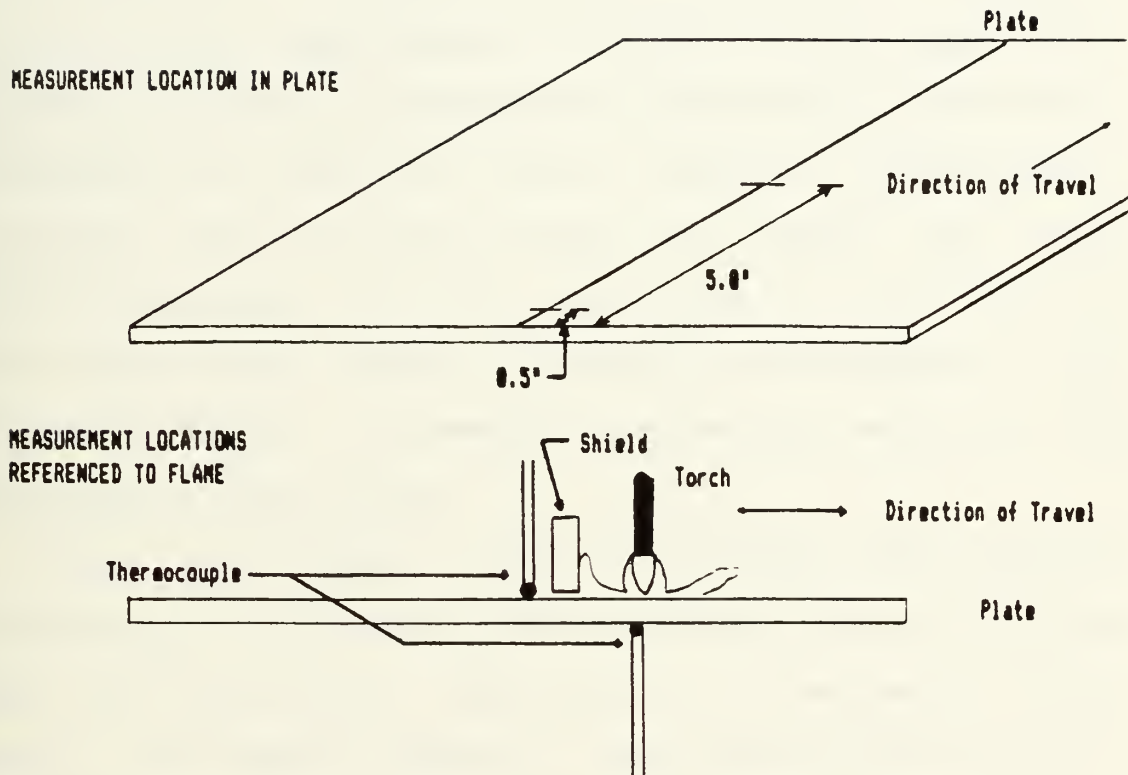
FIGURE 3.2 TEMPERATURE MEASUREMENTS: LOCATION AND TECHNIQUE

CHAPTER 4
EXPERIMENTAL RESULTS

4.1 General

The actual data obtained from the three experiments performed described the bending as a function of position at various points along the line of heating in each plate. This data could be used directly to examine the "edge effect" associated with the location of the heat source within each plate. However, to study the influence of plate dimensions (length) on bending, an average deflection was calculated for each plate.

These experimental results indicated that bending varied within the plate, generally being less at the plate edges, and that bending decreased with decreasing plate length. The results supporting these observations are presented and discussed in the remainder of this chapter.

Additionally, the results of the temperature measurements taken at the top and bottom surfaces of representative plates for each heat input, are presented in Table 4.1.

4.2 Angular deflection vs Plate length

The experimental results relating to this behavior are presented in **Table 4.2**, in which the angular deflections given represent   plate averages.   The general pattern observed in each case is the same, that of decreasing bending with decreasing plate length. The scatter exhibited by the data is attributed primarily to existing residual stress distributions in the plates themselves, which could differ widely from plate to plate.   As commented in **Table 3.2** of the previous chapter, Hot-rolled plates were used, to which different cutting and edging techniques were applied to the different plates by the commercial supplier.

To further increase the data available for analysis and to examine possible behavior variations with a different type of heat source and different plate thicknesses, LCDR W. H. Luebke graciously permitted measurements to be taken of plates line heated in experiments performed in the course of his investigation.[6]   These experiments were conducted using a variable power laser as the heat source, operated at power levels and travel speeds comparable to those used in the experiments presented in this thesis. Luebke's experiments examined the behavior of steel plates with thicknesses of 3/8", 1/2", and 5/8", and lengths of 1", 10", and 20".

In both cases, laser and flame line heating, the same width (18") plate was used. This data is summarized in **Table 4.3**.

The results obtained from both the flame and laser experiments served as basis for parametric studies performed in **Chapter 5**.

4.3 Angular Deflection vs. In Plate Location

The data used to determine the bending along each grid line for each plate of the first follow-on experiment, is fully detailed in **Tables 4.4** through **4.6**. Additionally, the data obtained in the cutting of the largest plate of the preliminary experiment is given in **Tables 4.7** and **4.8**. In both sets of data, the resulting angular deflections show variations in bending through the respective plates. However, the results of the follow-on experiment exhibit considerable scatter, particularly for the large plates, masking in part any general trends. To identify such trends, both a linear and quadratic least square fit was performed on the data for each plate; the results of which are presented and discussed in **Chapter 5**. The tabulated results for the cut plate, however, do provide a very clear pattern. The bending of the outer strips is definitely reduced, with an overall increase in bending between the strips when viewed in the same direction of travel as the

heat source. This reduced bending at the edges was expected
from the discussion of secondary factors; however the
overall increase in deflection along the length of the plate
was not. This implied asymmetric behavior with respect to
the plate center. A possible explanation of which is given
in the next chapter. It is noted that the linear and
quadratic least square fits result in curves presenting
similar behavior to that of the cut plate in two of the four
cases considered.

TORCH/TIP USED	PLATE LOCATION	THERMOCOUPLE READING (mV)	TEMPERATURE (deg C)
AIRCO TYPE 800			
# 7 TIP	PLATE NEAR EDGE (0.5" FROM EDGE)		
	TOP SURFACE	40.0*	1000.0*
	BOTTOM SURFACE	28.9	586.2
	PLATE INTERIOR (5.0" FROM EDGE)		
	TOP SURFACE	40.0*	1000.0*
	BOTTOM SURFACE	19.0	461.7
PUROX TYPE W-200			
# 6 TIP	PLATE NEAR EDGE: (0.5" FROM EDGE)		
	TOP SURFACE	21.0	500.6
	BOTTOM SURFACE	9.0	221.3
	PLATE INTERIOR: (5.0" FROM EDGE)		
	TOP SURFACE	21.0	500.6
	BOTTOM SURFACE	8.5	209.0

* NOTE: THESE READINGS HAD TO BE ESTIMATED, APPROXIMATE TEMPERATURE DETERMINED WITH TEMP-STICK CRAYONS

TABLE 4.1 TEMPERATURE MEASUREMENTS

SUMMARY OF ANGULAR DEFLECTIONS OBTAINED BY LASER LINE HEATING

LASER POWER	P / √v	DIMENSIONS	THETAt (rads)
6 Kw	2.0	1" x 18" x 3/8"	0.0049
		18" x 18" x 3/8"	0.0037
		20" x 18" x 3/8"	0.0067
6 Kw	2.5	1" x 18" x 3/8"	0.0028
		18" x 18" x 3/8"	0.0158
		20" x 18" x 3/8"	0.0178
5 Kw	1.0	1" x 18" x 1/2"	0.0014
		18" x 18" x 1/2"	0.0147
		20" x 18" x 1/2"	0.0300
5 Kw	2.5	1" x 18" x 1/2"	0.0000
		18" x 18" x 1/2"	0.0324
		20" x 18" x 1/2"	0.0260
8 Kw	3.0	1" x 18" x 5/8"	0.0032
		18" x 18" x 5/8"	0.0260
		20" x 18" x 5/8"	0.0324
8 Kw	3.5	1" x 18" x 5/8"	0.0034
		18" x 18" x 5/8"	0.0262
		20" x 18" x 5/8"	0.0300

TABLE 1.3

ANGULAR DEFLECTIONS BY PLATE - LASER HEATING

SUMMARY OF ANGULAR DEFLECTIONS OBTAINED BY FLAME LINE HEATING

EXPERIMENT	PLATE #	DIMENSIONS	THETAt (rads)
PRELIMINARY	0-1	3" x 18" x 3/8"	0.0127
	0-2	6" x 18" x 3/8"	0.0179
P / √v = 2.5	0-3	9" x 18" x 3/8"	0.0216
	0-4	12" x 18" x 3/8"	0.0248
FOLLOW-ON #1	1-1	3" x 18" x 3/8"	0.0045
	1-2	6" x 18" x 3/8"	0.0098
	1-3	9" x 18" x 3/8"	0.0092
P / √v = 1.5	1-4	12" x 18" x 3/8"	0.0080
	1-5	15" x 18" x 3/8"	0.0084
	1-6	18" x 18" x 3/8"	0.0098
FOLLOW-ON #2	2-1	3" x 18" x 3/8"	0.0079
	2-2	6" x 18" x 3/8"	0.0219
	2-3	9" x 18" x 3/8"	0.0221
P / √v = 2.5	2-4	12" x 18" x 3/8"	0.0390
	2-5	15" x 18" x 3/8"	0.0334
	2-6	18" x 18" x 3/8"	0.0399

TABLE 1.2

ANGULAR DEFLECTIONS BY PLATE - FLAME HEATING

PLATE #	DIMENSIONS	GRID LINE #	X	OUT OF PLANE DISTORTION Y							ANGULAR DEFLECTION -DEFLECTION ANGLES (rad)-		
				0.0	3.0	6.0	9.0	12.0	15.0	18.0	A1i	A2i	THETAi
1-1	3" X 18" X 3/8"	1	1.5	0.000	0.004	0.007	0.003	-0.001	-0.002	0.000	0.0004	-0.0003	-0.0007
			AVERAGE	0.000	0.004	0.007	0.003	-0.001	-0.002	0.000	0.0004	-0.0003	-0.0007
1-2	6" X 18" X 3/8"	1	1.5	-0.004	-0.043	-0.072	-0.087	-0.066	-0.032	0.000	-0.0093	0.0098	0.0191
		2	3.0	-0.006	-0.040	-0.068	-0.085	-0.066	-0.035	0.000	-0.0094	0.0087	0.0182
		3	4.5	0.002	-0.039	-0.069	-0.087	-0.075	-0.043	-0.028	-0.0099	0.0078	0.0177
			AVERAGE	-0.001	-0.041	-0.078	-0.086	-0.069	-0.037	-0.009	-0.0095	0.0088	0.0183
1-3	9" X 18" X 3/8"	1	1.5	-0.043	-0.048	-0.047	-0.051	-0.048	-0.029	0.000	-0.0010	0.0057	0.0068
		2	3.0	-0.034	-0.034	-0.041	-0.046	-0.041	-0.021	0.006	-0.0014	0.0059	0.0073
		3	4.5	-0.030	-0.032	-0.040	-0.045	-0.039	-0.015	0.011	-0.0010	0.0064	0.0082
		4	6.0	-0.034	-0.033	-0.042	-0.046	-0.039	-0.015	0.016	-0.0015	0.0078	0.0085
		5	7.5	-0.036	-0.037	-0.047	-0.050	-0.043	-0.014	0.018	-0.0015	0.0076	0.0091
			AVERAGE	-0.035	-0.035	-0.043	-0.048	-0.042	-0.019	0.010	-0.0015	0.0065	0.0080
1-4	12" X 18" X 3/8"	1	1.5	0.032	0.017	0.002	-0.007	-0.018	-0.007	0.000	-0.0044	0.0008	0.0052
		2	3.0	0.032	0.022	0.000	-0.002	-0.004	0.000	0.008	-0.0040	0.0011	0.0051
		3	4.5	0.032	0.020	0.007	0.000	0.002	0.004	0.015	-0.0036	0.0017	0.0053
		4	6.0	0.032	0.017	0.006	-0.001	0.002	0.006	0.019	-0.0037	0.0023	0.0059
		5	7.5	0.021	0.011	0.001	-0.004	0.003	0.005	0.020	-0.0028	0.0026	0.0054
		6	9.0	0.012	0.004	-0.005	-0.009	0.006	0.003	0.016	-0.0024	0.0032	0.0056
		7	10.5	0.002	-0.004	-0.011	-0.011	0.011	0.000	0.014	-0.0015	0.0031	0.0046
			AVERAGE	0.023	0.012	0.001	-0.005	0.002	0.002	0.014	-0.0032	0.0021	0.0053
1-5	15" X 18" X 3/8"	1	1.5	0.009	0.005	0.008	0.010	0.007	0.014	0.000	0.0002	0.0008	0.0010
		2	3.0	0.012	0.007	0.012	0.012	0.009	0.017	0.001	0.0002	0.0008	0.0010
		3	4.5	0.015	0.010	0.013	0.015	0.011	0.019	0.002	0.0001	0.0010	0.0011
		4	6.0	0.016	0.010	0.014	0.016	0.012	0.022	0.003	0.0001	0.0010	0.0011
		5	7.5	0.023	0.016	0.029	0.028	0.013	0.024	0.004	0.0001	0.0012	0.0014
		6	9.0	0.024	0.011	0.015	0.018	0.014	0.022	0.005	0.0005	0.0010	0.0006
		7	10.5	0.024	0.011	0.016	0.017	0.014	0.022	0.007	0.0007	0.0009	0.0002
		8	12.0	0.024	0.010	0.014	0.015	0.012	0.020	0.003	0.0003	0.0007	0.0002
		9	13.5	0.025	0.010	0.012	0.013	0.010	0.019	0.003	0.0003	0.0007	0.0004
			AVERAGE	0.019	0.012	0.015	0.015	0.011	0.020	0.003	0.0002	0.0009	0.0007
1-6	18" X 18" X 3/8"	1	1.5	-0.010	-0.007	-0.008	-0.010	-0.010	-0.008	0.000	0.0000	0.0011	0.0011
		2	3.0	-0.012	-0.010	-0.011	-0.011	-0.011	-0.008	0.001	0.0001	0.0003	0.0003
		3	4.5	-0.012	-0.010	-0.012	-0.012	-0.010	-0.010	0.004	0.0001	0.0010	0.0010
		4	6.0	-0.008	-0.006	-0.009	-0.011	-0.011	-0.008	0.006	0.0004	0.0006	0.0010
		5	7.5	-0.010	-0.008	-0.012	-0.013	-0.015	-0.011	0.010	0.0004	0.0004	0.0009
		6	9.0	-0.010	-0.009	-0.013	-0.014	-0.016	-0.011	0.005	0.0004	0.0008	0.0012
		7	10.5	-0.010	-0.010	-0.014	-0.014	-0.015	-0.011	0.007	0.0005	0.0011	0.0015
		8	12.0	-0.011	-0.010	-0.014	-0.011	-0.010	-0.011	0.007	0.0001	0.0005	0.0007
		9	13.5	-0.011	-0.010	-0.012	-0.012	-0.015	-0.011	0.008	0.0005	0.0009	0.0014
		10	15.0	-0.012	-0.011	-0.013	-0.012	-0.016	-0.011	0.008	0.0004	0.0006	0.0006
		11	16.5	-0.011	-0.009	-0.012	-0.012	-0.017	-0.010	0.007	0.0003	0.0008	0.0012
			AVERAGE	-0.011	-0.009	-0.012	-0.012	-0.015	-0.010	0.007	0.0003	0.0007	0.0010

TABLE 4.4 PRE-HEATING DATA AND CALCULATED ANGULAR DEFLECTIONS - FOLLOW-ON EXPERIMENT # 1

PLATE #	DIMENSIONS	GRID LINE #	X	OUT OF PLANE DISTORTION			Y				ANGULAR DEFLECTION :-DEFLECTION ANGLES (rad)-:		
				0.0	3.0	6.0	9.0	12.0	15.0	18.0	A1f	A2f	THETAf
1-1	3" X 18" X 3/8"	1	1.5	-0.035	-0.032	-0.030	-0.035	-0.025	-0.015	0.000	0.0001	0.0036	0.0036
			AVERAGE	-0.035	-0.032	-0.030	-0.035	-0.025	-0.015	0.000	0.0001	0.0036	0.0036
1-2	6" X 18" X 3/8"	1	1.5	-0.235	-0.250	-0.254	-0.242	-0.172	-0.094	0.000	-0.0000	0.0271	0.0200
		2	3.0	-0.233	-0.248	-0.252	-0.244	-0.174	-0.097	-0.011	-0.0012	0.0262	0.0274
		3	4.5	-0.232	-0.250	-0.256	-0.247	-0.181	-0.099	-0.026	-0.0017	0.0248	0.0265
			AVERAGE	-0.233	-0.249	-0.254	-0.244	-0.176	-0.098	-0.012	-0.0013	0.0261	0.0273
1-3	9" X 18" X 3/8"	1	1.5	-0.140	-0.132	-0.137	-0.138	-0.106	-0.050	0.000	0.0000	0.0154	0.0154
		2	3.0	-0.132	-0.129	-0.135	-0.136	-0.102	-0.053	0.000	-0.0006	0.0168	0.0166
		3	4.5	-0.130	-0.126	-0.133	-0.135	-0.100	-0.048	0.013	-0.0007	0.0165	0.0173
		4	6.0	-0.130	-0.128	-0.135	-0.136	-0.104	-0.047	0.017	-0.0008	0.0171	0.0179
		5	7.5	-0.133	-0.131	-0.139	-0.141	-0.104	-0.058	0.011	-0.0011	0.0175	0.0186
			AVERAGE	-0.133	-0.129	-0.136	-0.137	-0.102	-0.051	0.011	-0.0006	0.0165	0.0171
1-4	12" X 18" X 3/8"	1	1.5	-0.038	-0.048	-0.064	-0.074	-0.056	-0.032	0.000	-0.0049	0.0002	0.0131
		2	3.0	-0.026	-0.045	-0.060	-0.068	-0.050	-0.024	0.005	-0.0048	0.0003	0.0131
		3	4.5	-0.026	-0.045	-0.059	-0.067	-0.047	-0.028	0.015	-0.0046	0.0091	0.0137
		4	6.0	-0.032	-0.046	-0.060	-0.066	-0.045	-0.019	0.019	-0.0039	0.0094	0.0133
		5	7.5	-0.038	-0.049	-0.061	-0.069	-0.047	-0.017	0.020	-0.0035	0.0099	0.0134
		6	9.0	-0.047	-0.055	-0.068	-0.073	-0.048	-0.018	0.020	-0.0030	0.0103	0.0133
		7	10.5	-0.055	-0.065	-0.073	-0.076	-0.050	-0.020	0.020	-0.0024	0.0106	0.0130
			AVERAGE	-0.036	-0.050	-0.064	-0.071	-0.049	-0.021	0.014	-0.0039	0.0094	0.0133
1-5	15" X 18" X 3/8"	1	1.5	0.006	0.056	0.034	0.012	0.007	0.015	0.000	0.0001	-0.0009	0.0072
		2	3.0	0.008	0.057	0.036	0.013	0.008	0.016	0.000	-0.0002	-0.0010	0.0072
		3	4.5	0.090	0.058	0.036	0.014	0.008	0.017	0.001	-0.0003	-0.0011	0.0072
		4	6.0	0.090	0.059	0.036	0.013	0.009	0.017	0.001	-0.0005	-0.0009	0.0075
		5	7.5	0.090	0.058	0.035	0.012	0.010	0.017	0.000	-0.0005	-0.0009	0.0077
		6	9.0	0.009	0.058	0.032	0.011	0.009	0.016	0.000	-0.0007	-0.0008	0.0079
		7	10.5	0.009	0.058	0.032	0.011	0.009	0.016	0.001	-0.0007	-0.0009	0.0078
		8	12.0	0.009	0.058	0.031	0.010	0.006	0.016	0.001	-0.0009	-0.0006	0.0083
		9	13.5	0.093	0.053	0.034	0.011	0.008	0.014	0.000	-0.0002	-0.0005	0.0087
			AVERAGE	0.009	0.057	0.034	0.011	0.008	0.016	0.000	-0.0006	-0.0009	0.0077
1-6	18" X 18" X 3/8"	1	1.5	0.119	0.091	0.052	0.010	0.011	0.007	0.000	-0.0122	-0.0011	0.0111
		2	3.0	0.118	0.098	0.051	0.018	0.009	0.004	-0.007	-0.0113	-0.0027	0.0086
		3	4.5	0.118	0.091	0.050	0.010	0.007	0.006	-0.005	-0.0114	-0.0023	0.0098
		4	6.0	0.124	0.096	0.054	0.018	0.001	0.006	-0.006	-0.0120	-0.0022	0.0090
		5	7.5	0.125	0.095	0.051	0.015	0.008	0.003	-0.004	-0.0125	-0.0021	0.0104
		6	9.0	0.125	0.094	0.052	0.018	0.017	0.004	-0.002	-0.0122	-0.0028	0.0102
		7	10.5	0.124	0.094	0.052	0.018	0.007	0.008	0.000	-0.0128	-0.0018	0.0102
		8	12.0	0.125	0.094	0.054	0.018	0.008	0.005	-0.002	-0.0128	-0.0021	0.0099
		9	13.5	0.126	0.094	0.054	0.018	0.008	0.004	-0.004	-0.0123	-0.0021	0.0102
		10	15.0	0.127	0.091	0.053	0.018	0.008	0.005	-0.002	-0.0121	-0.0022	0.0100
		11	16.5	0.127	0.093	0.052	0.019	0.007	0.005	-0.003	-0.0128	-0.0021	0.0099
			AVERAGE	0.123	0.093	0.052	0.017	0.007	0.005	-0.003	-0.0128	-0.0021	0.0099

TABLE 4.5 POST-HEATING DATA AND CALCULATED ANGULAR DEFLECTIONS - FOLLOW-ON EXPERIMENT # 1

PLATE #	DIMENSIONS	GRID LINE #	X	OUT OF PLANE DISTORTION — Y							ANGULAR DEFLECTION — DEFLECTION ANGLES (rad)		
				0.0	3.0	6.0	9.0	12.0	15.0	18.0	A1t	A2t	THETAt
1-1	3" X 18" X 3/8"	1	1.5	-0.035	-0.036	-0.037	-0.038	-0.024	-0.013	0.000	-0.0003	0.0042	0.0045
			AVERAGE	-0.035	-0.036	-0.037	-0.038	-0.024	-0.013	0.000	-0.0003	0.0042	0.0045
1-2	6" X 18" X 3/8"	1	1.5	-0.231	-0.207	-0.182	-0.155	-0.106	-0.052	0.000	0.0004	0.0173	0.0089
		2	3.0	-0.233	-0.208	-0.184	-0.159	-0.108	-0.052	-0.003	0.0002	0.0175	0.0093
		3	4.5	-0.234	-0.211	-0.187	-0.168	-0.106	-0.056	-0.006	0.0002	0.0171	0.0089
			AVERAGE	-0.233	-0.209	-0.184	-0.158	-0.107	-0.053	-0.003	0.0003	0.0173	0.0090
1-3	9" X 18" X 3/8"	1	1.5	-0.097	-0.092	-0.090	-0.087	-0.058	-0.029	0.000	0.0011	0.0097	0.0086
		2	3.0	-0.098	-0.095	-0.091	-0.098	-0.061	-0.032	0.002	0.0010	0.0102	0.0093
		3	4.5	-0.100	-0.094	-0.093	-0.098	-0.061	-0.033	0.002	0.0010	0.0101	0.0091
		4	6.0	-0.096	-0.095	-0.093	-0.091	-0.061	-0.032	0.001	0.0007	0.0101	0.0094
		5	7.5	-0.097	-0.094	-0.092	-0.098	-0.060	-0.036	0.000	0.0005	0.0099	0.0095
			AVERAGE	-0.097	-0.094	-0.092	-0.098	-0.060	-0.032	0.001	0.0008	0.0100	0.0092
1-4	12" X 18" X 3/8"	1	1.5	-0.062	-0.065	-0.066	-0.067	-0.046	-0.025	0.000	-0.0005	0.0074	0.0079
		2	3.0	-0.058	-0.067	-0.066	-0.067	-0.046	-0.024	-0.003	-0.0008	0.0071	0.0080
		3	4.5	-0.058	-0.065	-0.066	-0.067	-0.045	-0.024	0.000	-0.0009	0.0074	0.0083
		4	6.0	-0.064	-0.063	-0.066	-0.065	-0.044	-0.025	-0.002	-0.0002	0.0071	0.0073
		5	7.5	-0.059	-0.060	-0.062	-0.064	-0.042	-0.022	-0.003	-0.0007	0.0073	0.0080
		6	9.0	-0.059	-0.059	-0.063	-0.064	-0.039	-0.021	0.000	-0.0006	0.0071	0.0077
		7	10.5	-0.057	-0.061	-0.062	-0.065	-0.044	-0.028	0.004	-0.0008	0.0071	0.0081
			AVERAGE	-0.060	-0.063	-0.065	-0.066	-0.044	-0.023	0.001	-0.0007	0.0073	0.0080
1-5	15" X 18" X 3/8"	1	1.5	0.077	0.051	0.026	0.002	0.000	0.001	0.000	-0.0003	-0.0002	0.0082
		2	3.0	0.076	0.050	0.024	0.001	-0.001	0.001	-0.001	-0.0004	-0.0030	0.0082
		3	4.5	0.075	0.048	0.023	0.001	-0.003	0.002	-0.002	-0.0004	-0.0031	0.0084
		4	6.0	0.074	0.049	0.022	0.003	-0.003	0.005	-0.002	-0.0006	0.0000	0.0086
		5	7.5	0.067	0.042	0.007	0.000	-0.003	0.007	-0.005	-0.0007	0.0004	0.0090
		6	9.0	0.066	0.040	0.020	0.007	0.005	0.005	-0.005	-0.0002	0.0002	0.0094
		7	10.5	0.065	0.047	0.016	0.006	0.005	0.006	-0.007	-0.0001	-0.0001	0.0084
		8	12.0	0.065	0.048	0.018	0.006	0.003	0.004	-0.002	-0.0001	-0.0004	0.0085
		9	13.5	0.068	0.043	0.019	0.005	0.004	0.005	-0.003	-0.0003	0.0002	0.0083
			AVERAGE	0.070	0.047	0.019	0.004	0.003	0.004	-0.003	-0.0003	-0.0001	0.0084
1-6	18" X 18" X 3/8"	1	1.5	0.129	0.090	0.060	0.028	0.021	0.015	0.000	-0.0122	-0.0022	0.0100
		2	3.0	0.130	0.100	0.062	0.029	0.020	0.014	0.001	-0.0114	-0.0030	0.0104
		3	4.5	0.130	0.101	0.062	0.030	0.022	0.016	-0.001	-0.0113	-0.0033	0.0100
		4	6.0	0.132	0.102	0.063	0.029	0.012	0.014	0.000	-0.0116	-0.0028	0.0088
		5	7.5	0.135	0.103	0.063	0.028	0.023	0.014	0.006	-0.0128	-0.0025	0.0095
		6	9.0	0.136	0.104	0.065	0.031	0.024	0.015	0.006	-0.0116	-0.0028	0.0090
		7	10.5	0.134	0.104	0.064	0.032	0.023	0.017	0.005	-0.0115	-0.0029	0.0086
		8	12.0	0.136	0.104	0.068	0.029	0.023	0.016	0.005	-0.0119	-0.0026	0.0093
		9	13.5	0.137	0.104	0.066	0.031	0.026	0.015	0.004	-0.0119	-0.0031	0.0088
		10	15.0	0.138	0.104	0.066	0.038	0.025	0.016	0.005	-0.0121	-0.0028	0.0093
		11	16.5	0.130	0.102	0.065	0.033	0.024	0.017	0.006	-0.0117	-0.0029	0.0000
			AVERAGE	0.134	0.102	0.064	0.029	0.022	0.015	0.003	-0.0118	-0.0028	0.0098

TABLE 4.6   EFFECT DUE TO LINE HEATING - FOLLOW-ON EXPERIMENT # 1

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GRID LINE #	X	OUT OF PLANE DISTORTION								ANGULAR DEFLECTION	
		0.00	1.50	4.50	7.50	10.50	13.50	16.50	18.00	THETAt	CHANGE
		INITIAL PLATE									
8	0.75	0.178	0.152	0.086	0.013	-0.023	-0.012	-0.005	-0.003	0.0240	0.0000
7	2.25	0.173	0.140	0.083	0.014	-0.027	-0.018	-0.012	-0.011	0.0236	0.0000
6	3.75	0.173	0.140	0.085	0.018	-0.030	-0.020	-0.019	-0.018	0.0233	0.0000
5	5.25	0.172	0.149	0.000	0.009	-0.030	-0.027	-0.025	-0.020	0.0229	0.0000
4	6.75	0.178	0.140	0.078	0.018	-0.031	-0.038	-0.020	-0.028	0.0238	0.0000
3	8.25	0.173	0.156	0.079	0.007	-0.032	-0.030	-0.031	-0.032	0.0220	0.0000
2	9.75	0.176	0.156	0.081	0.018	-0.030	-0.030	-0.031	-0.034	0.0223	0.0000
1	11.25	0.166	0.160	0.082	0.018	-0.029	-0.030	-0.032	-0.034	0.0233	0.0000
		FIRST CUT									
8	0.75	0.180	0.156	0.088	0.017	-0.022	-0.012	-0.005	-0.004	0.0253	0.0005
7	2.25	0.185	0.155	0.087	0.016	-0.026	-0.018	-0.013	-0.011	0.0246	0.0011
6	3.75	0.185	0.155	0.085	0.015	-0.028	-0.022	-0.019	-0.015	0.0244	0.0011
5	5.25	0.186	0.156	0.086	0.014	-0.030	-0.026	-0.023	-0.021	0.0242	0.0014
4	6.75	0.189	0.158	0.088	0.009	-0.029	-0.025	-0.025	-0.023	0.0247	0.0017
3	8.25	0.192	0.163	0.085	0.013	-0.030	-0.027	-0.026	-0.025	0.0249	0.0020
2	9.75	0.198	0.165	0.090	0.012	-0.030	-0.025	-0.024	-0.024	0.0251	0.0028
1	11.25	0.123	0.100	0.050	-0.001	-0.034	-0.033	-0.032	-0.034	0.0167	-0.0066
		SECOND CUT									
8	0.75	0.193	0.161	0.094	0.024	-0.018	-0.012	-0.005	-0.005	0.0244	-0.0009
7	2.25	0.191	0.160	0.091	0.018	-0.023	-0.025	-0.015	-0.012	0.0248	0.0002
6	3.75	0.186	0.156	0.087	0.014	-0.033	-0.025	-0.019	-0.017	0.0252	0.0007
5	5.25	0.188	0.157	0.088	0.011	-0.033	-0.028	-0.023	-0.021	0.0253	0.0011
4	6.75	0.191	0.157	0.088	0.014	-0.032	-0.028	-0.026	-0.023	0.0247	0.0001
3	8.25	0.195	0.163	0.098	0.014	-0.030	-0.025	-0.025	-0.025	0.0250	0.0001
2	9.75	0.169	0.140	0.075	0.010	-0.024	-0.022	-0.024	-0.024	0.0212	-0.0038
1	11.25	0.123	0.100	0.050	-0.001	-0.034	-0.033	-0.032	-0.034	0.0167	0.0000
		THIRD CUT									
8	0.75	0.222	0.187	0.116	0.040	-0.006	-0.002	-0.000	0.000	0.0250	0.0006
7	2.25	0.221	0.187	0.114	0.036	-0.010	-0.007	-0.006	-0.006	0.0253	0.0005
6	3.75	0.224	0.190	0.114	0.036	-0.011	-0.011	-0.009	-0.009	0.0255	0.0003
5	5.25	0.227	0.192	0.115	0.037	-0.014	-0.013	-0.012	-0.012	0.0257	0.0004
4	6.75	0.231	0.195	0.121	0.036	-0.012	-0.011	-0.012	-0.012	0.0259	0.0012
3	8.25	0.185	0.153	0.089	0.015	-0.026	-0.023	-0.025	-0.025	0.0226	-0.0023
2	9.75	0.169	0.140	0.075	0.010	-0.024	-0.022	-0.024	-0.024	0.0212	0.0000
1	11.25	0.123	0.100	0.050	-0.001	-0.034	-0.033	-0.032	-0.034	0.0167	0.0000

TABLE 4.7 EFFECT OF SECTIONING PLATE 8-4: FIRST THROUGH THIRD CUT

GRID LINE #	X	OUT OF PLANE DISTORTION								ANGULAR DEFLECTION	
		Y								THETAt	CHANGE
		0.00	1.50	4.50	7.50	10.50	13.50	16.50	18.00		
						FOURTH CUT					
8	0.75	0.232	0.198	0.122	0.044	-0.003	-0.002	-0.002	-0.002	0.0253	0.0002
7	2.25	0.234	0.198	0.126	0.044	-0.005	-0.003	-0.002	-0.005	0.0253	0.0001
6	3.75	0.241	0.201	0.128	0.047	-0.003	-0.003	-0.003	-0.005	0.0255	-0.0004
5	5.25	0.250	0.210	0.135	0.052	-0.002	-0.001	-0.003	-0.002	0.0262	0.0004
4	6.75	0.200	0.164	0.087	0.015	-0.020	-0.024	-0.024	-0.025	0.0242	-0.0017
3	8.25	0.185	0.153	0.069	0.015	-0.026	-0.023	-0.025	-0.025	0.0226	0.0000
2	9.75	0.169	0.140	0.075	0.010	-0.024	-0.022	-0.024	-0.024	0.0212	0.0000
1	11.25	0.123	0.100	0.050	-0.001	-0.034	-0.033	-0.032	-0.034	0.0167	0.0000
						FIFTH CUT					
8	0.75	0.247	0.211	0.137	0.057	0.005	0.004	0.001	0.002	0.0244	-0.0009
7	2.25	0.250	0.214	0.135	0.056	0.001	0.002	-0.003	-0.006	0.0248	-0.0005
6	3.75	0.253	0.219	0.137	0.051	0.002	0.000	0.006	-0.007	0.0255	-0.0003
5	5.25	0.239	0.202	0.127	0.044	-0.001	-0.002	-0.002	-0.002	0.0258	0.0000
4	6.75	0.200	0.164	0.087	0.015	-0.020	-0.024	-0.024	-0.025	0.0242	0.0000
3	8.25	0.185	0.153	0.069	0.015	-0.026	-0.023	-0.025	-0.025	0.0226	0.0000
2	9.75	0.169	0.140	0.075	0.010	-0.024	-0.022	-0.024	-0.024	0.0212	0.0000
1	11.25	0.123	0.100	0.050	-0.001	-0.034	-0.033	-0.032	-0.034	0.0167	0.0000
						SIXTH CUT					
8	0.75	0.235	0.200	0.131	0.052	0.003	0.005	0.004	0.003	0.0244	-0.0001
7	2.25	0.240	0.205	0.129	0.054	0.004	0.005	0.003	0.002	0.0246	-0.0001
6	3.75	0.241	0.206	0.135	0.047	-0.005	-0.006	-0.006	-0.007	0.0259	-0.0004
5	5.25	0.239	0.202	0.127	0.044	-0.001	-0.002	-0.002	-0.002	0.0258	0.0000
4	6.75	0.200	0.164	0.087	0.015	-0.020	-0.024	-0.024	-0.025	0.0242	0.0000
3	8.25	0.185	0.153	0.069	0.015	-0.026	-0.023	-0.025	-0.025	0.0226	0.0000
2	9.75	0.169	0.140	0.075	0.010	-0.024	-0.022	-0.024	-0.024	0.0212	0.0000
1	11.25	0.123	0.100	0.050	-0.001	-0.034	-0.033	-0.032	-0.034	0.0167	0.0000
						SEVENTH CUT					
8	0.75	0.246	0.213	0.140	0.065	0.012	0.011	0.007	0.003	0.0231	-0.0012
7	2.25	0.250	0.210	0.140	0.056	0.003	0.004	0.002	0.002	0.0254	0.0000
6	3.75	0.241	0.200	0.135	0.047	-0.005	-0.005	-0.006	-0.007	0.0259	0.0000
5	5.25	0.239	0.202	0.127	0.044	-0.001	-0.002	-0.002	-0.002	0.0258	0.0000
4	6.75	0.200	0.164	0.087	0.015	-0.020	-0.024	-0.024	-0.025	0.0242	0.0000
3	8.25	0.185	0.153	0.069	0.015	-0.026	-0.023	-0.025	-0.025	0.0226	0.0000
2	9.75	0.169	0.140	0.075	0.010	-0.024	-0.022	-0.024	-0.024	0.0212	0.0000
1	11.25	0.123	0.100	0.050	-0.001	-0.034	-0.033	-0.032	-0.034	0.0167	0.0000

TABLE 4.8 EFFECT OF SECTIONING PLATE 8-4: FOURTH THROUGH SEVENTH CUT

CHAPTER 5
ANALYSIS

5.1   General

The analysis of the results presented in the previous
chapter was divided into two parts: angular deflection vs.
plate length and angular deflection vs. in plate location
(or edge effects).   The approach taken when considering
bending vs. plate length was first to compare the results
with the already existing body of data and secondly, to ap-
propriate analytic models.   A parametric study was per-
formed, and parameter values related to already established
parameters describing plate bending.   The analysis of the
edge behavior, for which less existing data is available,
was less quantitative.   The observed bending behavior was
described in terms of a qualitative model, and an attempt to
determine the relative contribution of the two principal
factors made.

5.2   Angular Deflection vs. Plate Length

The analysis of the experimental data relating to this
behavior considered first the validity of the results, both
in the light of previous experimental data, and also with
established models for thermo-mechanical bending of plates.
The major source of existing experimental data used for com-
parison was the Phase I Report. In particular, the results
of the parametric study of $P/\sqrt{v}$ vs. θ_t. The angular deflec-
tions of the larger plates (1-6 and 2-6) corresponding to
the estimated values of $P/\sqrt{v}$ used in the two follow-on
experiments were superimposed on the existing graph of $P/\sqrt{v}$
vs. θ_t. **Figure 5.1** illustrates that these added data points
fall in a region between the curves depicting the behavior
of the 1/4" and 1/2" plates, exhibiting the same general
pattern. Additionally, the trend of increasing angular
deflection at a decreasing rate noted in the preliminary and
follow-on experiment, is very similar to that observed for
the line heated 12"x12"x1/2" plate ($P/\sqrt{v}=1.8$) depicted in
Figure 2.4, in which heating was along lines parallel to the
plate edge. The laser line heating data also exhibited a
similar trend. The major factor expected to influence this
behavior, as discussed in **Chapter 2**, is related to the prox-
imity of the plate edge. Decreasing the plate dimension in
the direction of heating reduces the amount of plate

material available to provide mechanical restraint and act
as a heat sink. So as plate length is reduced, the "edge
effects" should progressively dominate, reducing the amount
of bending in the same manner as outlined in the discussion
of the expected of reducing the plate width given in **Chapter
2.** The transition should be a gradual one.

In this case, the angular deflection observed in a
plate of a given length could possibly be determined based
on the distance from the plate center to the perpendicular
edge, i.e. by L/2. To examine this possibility, the results
of the line heated 1/2" plates (obtained under experimental
conditions that best approximated those specified in **Figure
2.4**), were plotted as a function of L/2. The curve so
obtained was then superimposed on **Figure 2.4.** The result,
depicted in **Figure 5.2,** is that the 2 curves essentially
coincide. The implications of this correspondence is that
the effect of varying the plate dimensions on bending is the
same for the length as for the width, and can be predicted
using the same curve that describes the bending as the line
of heating approaches a plate edge parallel to it. There-
fore, not only do the results appear to be in general agree-
ment with previous experimental data, but they also afford a
means of accounting for the plate dimensions in the plate
bending predictions, provided the curve is known for

that particular plate and value of $P/\sqrt{v}$. The parametric
studies discussed later in this section address how this
curve should relate to $P/\sqrt{v}$.

The heat deformation model was used to examine quanti-
tatively the expected large plate deflection with respect to
measured top and bottom plate temperatures. The results of
these calculations are summarized and compared to the exper-
imental results in **Table 5.1.** The model underestimates the
experimental values by a factor of approximately 2.5. This
was in fact predicted by Luebke, based on results obtained
during the course of his investigation. When this model was
applied in his studies of bending with variations in plate
thickness, it was noted that the model underestimated the
distortion for plates of thickness less than approximately
0.58" and overestimated the distortion for plates of greater
thickness, with the variation being a linear function of
thickness over the range studied. For a plate thickness of
3/8" a correction factor of 3.0 was determined. Therefore,
the corrected prediction and experimental results appear in
reasonable agreement.

In order to value narrow plate deflection, the analytic
model developed by Iwamura and Rybicki was considered. As
mentioned in **Chapter 2,** a serious attempt to develop a

computer program based on the model was undertaken. However, stability problems relating to the calculation of the mechanical strain at points along lines through the plate strip could not be adequately resolved. The integration of the stress estimating functions along the thickness of the plate appeared to produce alternating stable configurations primarily during the transition from heat up to cool down. This resulted in premature termination of the iterative procedure prior to the cool down phase. Results produced during the heat up phase of the computer simulation did appear to be in agreement with observed behavior for that point of the thermal cycle, specifically that of out-of-plane distortion and angular deflection. However, the inability to carry the calculations on through the cool down phase, did not permit useful quantitative output predicting the final distribution of residual stresses and resulting deflections to be obtained. A quantitative comparison with Iwamura and Rybicki's published results is not valid due to the inherent differences in the narrow plate modeled (5mm wide, 50mm thick, 500mm long) and the experimental procedures (differences in the application of heat, use of forced cooling) as compared to those presented here. In a qualitative sense, the angular deflections (both theoretical

and experimental) that they obtained, of 3.5 milli-radians does compare favorable with those obtained for the narrow plates (plates 0-1, 12.7 milli-rads; plate 1-1, 4.5 milli-rads; plate 2-1, 7.9 milli-rads).

Characterization of the behavior, in the form of a parametric study, was then undertaken. The behavior suggested a functional relationship of the form:

$$\theta_t(L) = A(1-e^{-BL})$$

where θ_t = total angular deflection,

L = the plate length.

Further, it was expected that both parameters, A and B, should be related to the value of $P/V\sqrt{V}$ corresponding to the experiment. Clearly, the limiting value of the relationship, A, would be the bending obtained in large plates for a given thickness, and so dependent on $P/V\sqrt{V}$ as expressed in the parametric studies of the Phase I Report. The decay constant, B, on the other hand could exhibit more dependence on velocity than provided by the parameter $P/V\sqrt{V}$.

The results of the parametric study are contained in the Appendix. The study included the preliminary experiment, the 2 follow-on experiments, as well as the laser line heating data. It was found to be more meaningful to average the data when possible, to reduce random effects. In particular, the data of the preliminary and second follow-on

experiments were averaged for the first four plates and the
laser line heating data was averaged for each of the 3
thickness of the plate considered. A summary of the parame-
ter values, A and B, obtained in each case is given in Table
5.2, accompanied by the corresponding power and speed data.
A preliminary comparison indicates that A does indeed depend
on $P/\sqrt{v}$ as previously described, and that B appears to be
proportional to P/v^n, with n in the range of 0.5 to 1.5.
Given the correspondence of deflective behavior with respect
to distance from both the parallel and perpendicular edges,
these parametric curves would then serve to predict bending
of a plate based on its dimensions and the location of the
line of heating.

5.3 Angular Deflection vs. In Plate Location--
Edge Effects

Given the significance of edge effects on the bending
of narrow plates, a more detailed study of these effects was
conducted. Commencing with the preliminary experiment, the
0-4 plate was cut as previously described to provide infor-
mation that could serve as a basis for such a study. The
most immediate result was that of the asymmetry in the
behavior. Differing from the previously discussed symmetric
behavior as heating along lines parallel to the plate edge

progresses from one edge to the other (neglecting any dif-
fering residual stress distributions already present), the
plate exhibits varying amounts of bending as the heat source
enters over and exits the corresponding perpendicular (to
the line of heating) edges. This variation was noted in the
majority of the plates studied, generally presenting the
same pattern. This pattern was of reduced bending at the
entrance edge of the heating source, increasing at an
increasing rate over a portion of the interior, then at
essentially constant rate, until approaching the far edge of
the plate where bending again decreased but not as much as
at the entrance. The angular deflection profile, graphed in
Figure 5.3, of the cut plate segments clearly illustrates
this asymmetry. A quadratic least squares fit resulted in a
very close correspondence. The plate strips had been cut
out sequentially, progressing in the same direction as the
heat source, with the first strip cut being centered about
the 11.25" grid line. The bending at this edge was approxi-
mately 75% of that at the far edge and 64% of the maximum
bending as measured in the free strips.

An attempt to verify similar behavior in other plates,
without the need to section each plate, led to performing
the linear and quadratic least squares curve fittings to the
data corresponding to the first follow-on experiment (plates

1-3 through 1-6; plates 1-1 and 1-2 were purposely excluded
as being too narrow) mentioned in **Chapter 4.** The graphs,
given in **Figures 5.4** through **5.7**, show the same behavior in
2 of the 4 plates. In all cases, the general trend is for
angular deflection to increase through the length of the
plate interior. The marked differences in the other two
plates (1-4 and 1-6) are expected to be principally due to
unrelieved stresses from the plate forming process, i.e.
rolling, press cut, etc. A qualitative description of the
expected mechanism producing this behavior is outlined in
the following paragraph.

Consider that an effective length exists, which defines
the plate material adjacent to that being heated by the
torch, that contributes directly to the restraint experi-
enced by the material under the torch. Further, that the
plate is divided into strips perpendicular to the direction
of travel, of width given by this effective length. As the
heat source moves over the first such strip, the bending
that will be produced is determined by the restraint pro-
vided by the adjacent interior strip and the thermal gra-
dient through the plate. Neglecting the thermal gradient
effect, this strip sees "one unit" of restraint as it begins
to deflect downward. As the heat source moves over the sec-
ond strip, the out board strip is starting to approach

zero deflection so this second strip is subject to essentially "2 units" of restraint (one from each adjacent strip). The third strip will likewise receive "2 units" of restraint from the second and fourth strips, but the first is now deflecting positively and will, through the second strip, provide an additional restraining moment. The third strip is then subjected to "3 units" of restraint. The sequence will repeat itself, with the restraint increasing for each successive strip, as previously heated strips cool and continue to deflect positively. Sufficiently into the plate interior, the first strips will have attained maximum positive deflection and so no longer act to increase the restraining moment; each successive strip is receiving the same amount of restraint. Finally, as the far edge is approached, the available restraint decreases. The amount of bending produced in each strip, varies in a likewise manner.

The reduced temperature gradient at the edges, neglected in the above, also influences the bending. This then leads to the question of the relative contribution of each effect; reduced material restraint and reduced temperature difference between the top and bottom surfaces of the plate. An attempt to provide an answer was made using the heat deformation model to estimate the thermal contribution,

and then comparing this with the total effect as seen in the cut plate. The representative temperature measurements taken at the near edge and interior, at both top and bottom surfaces were used to calculate the bending that would be expected due to temperature effects only. The results of the calculations, including intermediate values used in the heat deformation model, have been already given in **Table 5.1.** The ratio of the angular deflections corresponding to the temperatures at the edge to that of the temperatures of the plate interior was found to be 0.91. The ratios of the edge deflections to the maximum deflection for the cut plate were found to be 0.65 for the entrance edge and 0.92 for the exiting edge. This would indicate that the major effect at the entrance edge is that of the decreased material restraint, accounting for approximately 70% of the reduction in bending at that edge. At the exiting edge, the situation appears more complicated. The mechanical restraint due to the deforming geometry of the plate interior would appear to act in opposition to the combined effect, resulting in the 0.92 ratio. A similar calculation was performed for the temperatures recorded for the first follow-on experiment, with values also given in **Table 5.1.** However, comparisons with the recorded data for that experiment proved less

conclusive, with most plates showing ratios of edge deflec-
tion to interior deflection of approximately 0.95.

The inconclusive nature of these last comparisons could
be interpreted as an indication that other related secondary
factors, such as that of geometric restraint, are also con-
tributing to the behavior, particularly at the exiting edge.
Since the variations in bending being considered are rela-
tively small and close to the limits of the measurement
techniques used, further experiments focusing on this aspect
should be performed.   The increased data would reduce the
random influence of pre-stress conditions of the plate. If
the coefficients describing the quadratic behavior can be
determined for a plate then this would provide a means of
predicting the actual bending produced at any point on the
plate surface.

PLATE	LOCATION (TO EDGE)	TEMPERATURES (deg C)		INTERMEDIATE MODEL VARIABLES				ANGULAR DEFLECTIONS (rads)		
		TOP	BOTTOM	Th	Tr	c	ω	MODEL	CORRECTED	ACTUAL
1-6	0.5"	500.6	221.3	406.9	221.3	0.225"	0.775"	0.0042	0.0125	0.0090
P/√N=1.5	5.0"	500.6	209.0	414.1	209.0	0.225"	0.695"	0.0044	0.0131	0.0090
2-6	0.5"	1000.0	506.2	785.0	506.2	0.278"	1.149"	0.0124	0.0373	0.0399
P/√N=2.5	5.0"	1000.0	461.7	757.0	461.7	0.274"	1.176"	0.0136	0.0400	0.0399

TABLE 5.1 HEAT DEFORMATION MODEL PREDICTED DEFLECTIONS COMPARED TO EXPERIMENTAL RESULTS

EXPERIMENT	PLATE THICKNESS	THERMAL POWER	SPEED (in/min)	PARAMETER VALUES A	B	ERROR **	THETAt* (rads)	P/√n (n=0.5)	P/√n (n=1.5)
PRELIMINARY	3/8"	6.000	7.500	43.900	0.070	18.690	-.-	2.483	0.331
FOLLOW-ON # 1	3/8"	4.100	7.500	9.500	0.200	5.120	-.-	1.497	0.200
FOLLOW-ON # 2	3/8"	6.000	7.500	55.000	0.074	105.160	-.-	2.483	0.331
PRE AND FOL-ON # 2	3/8"	6.000	7.500	51.700	0.075	20.860	-.-	2.483	0.331
LASER # 1	3/8"	6.000	9.000	18.000	0.076	20.000	-.-	2.000	0.222
LASER # 2	3/8"	6.000	5.000	24.300	0.082	5.940	-.-	2.491	0.430
LASER # 1 AND # 2	3/8"	6.000	7.400	14.600	0.118	5.877	-.-	2.206	0.298
LASER # 3	1/2"	5.000	7.700	45.200	0.062	74.560	0.019	1.802	0.234
LASER # 4	1/2"	5.000	4.000	29.800	0.328	28.640	0.026	2.500	0.625
LASER # 3 AND # 4	1/2"	5.000	5.900	35.000	0.118	1.143	0.023	2.058	0.349
LASER # 5	5/8"	8.000	7.100	38.700	0.103	3.468	0.014	3.002	0.423
LASER # 6	5/8"	8.000	5.200	32.300	0.157	2.070	0.019	3.500	0.675
LASER # 5 AND # 6	5/8"	8.000	6.200	36.030	0.121	2.800	0.017	3.213	0.518

NOTES: * ESTIMATED VALUES FROM EXISTING P/√V vs THETAt CURVES
       ** SUM OF THE SQUARES OF THE DIFFERENCE OF THE DATA AND THE CURVE

TABLE 5.2 RESULTS OF PARAMETRIC STUDY
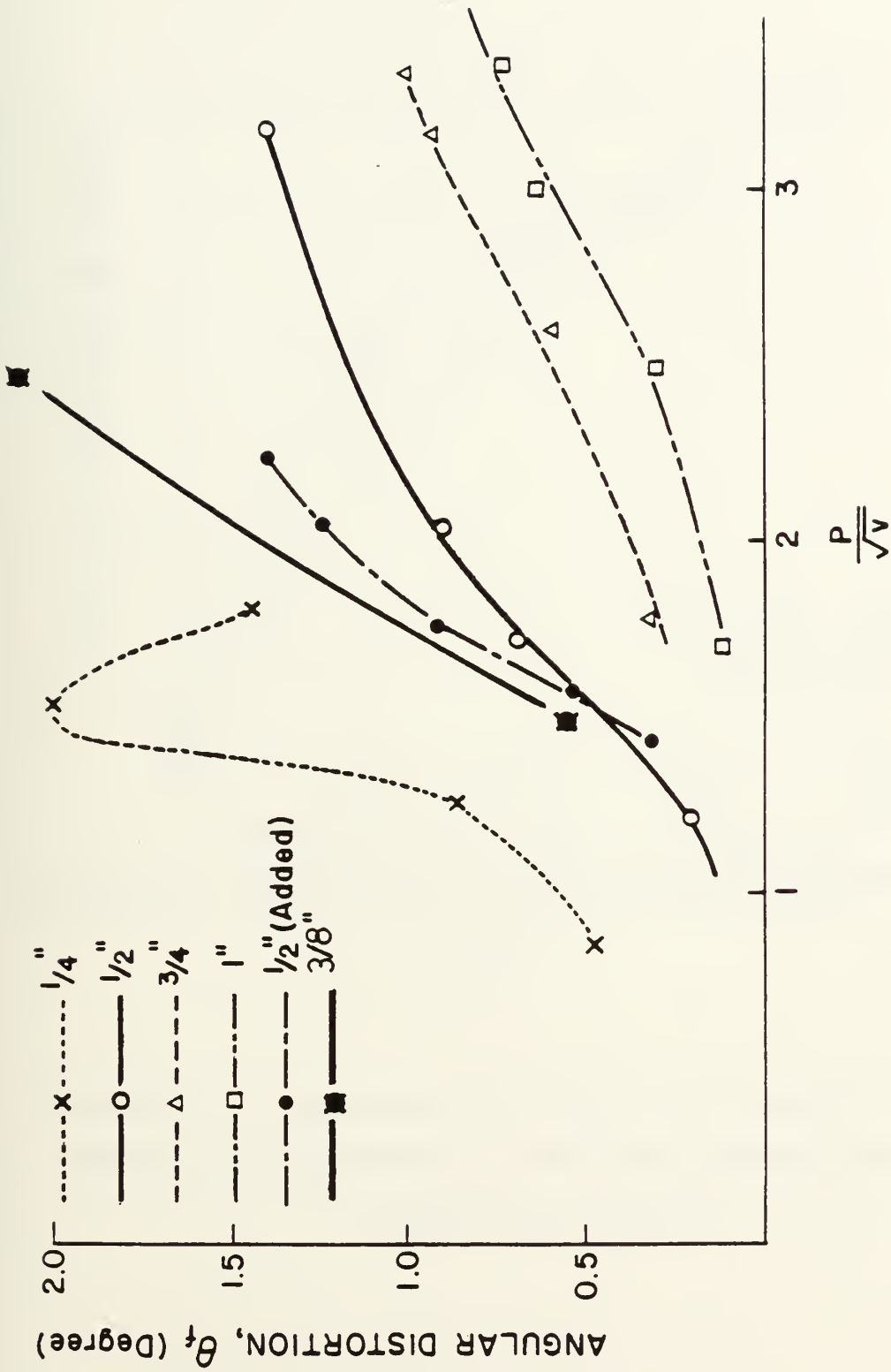
FIGURE 5.1 ADDITION OF 3/8" PLATE FLAME HEATING DATA TO EXISTING PARAMETRIC CURVES
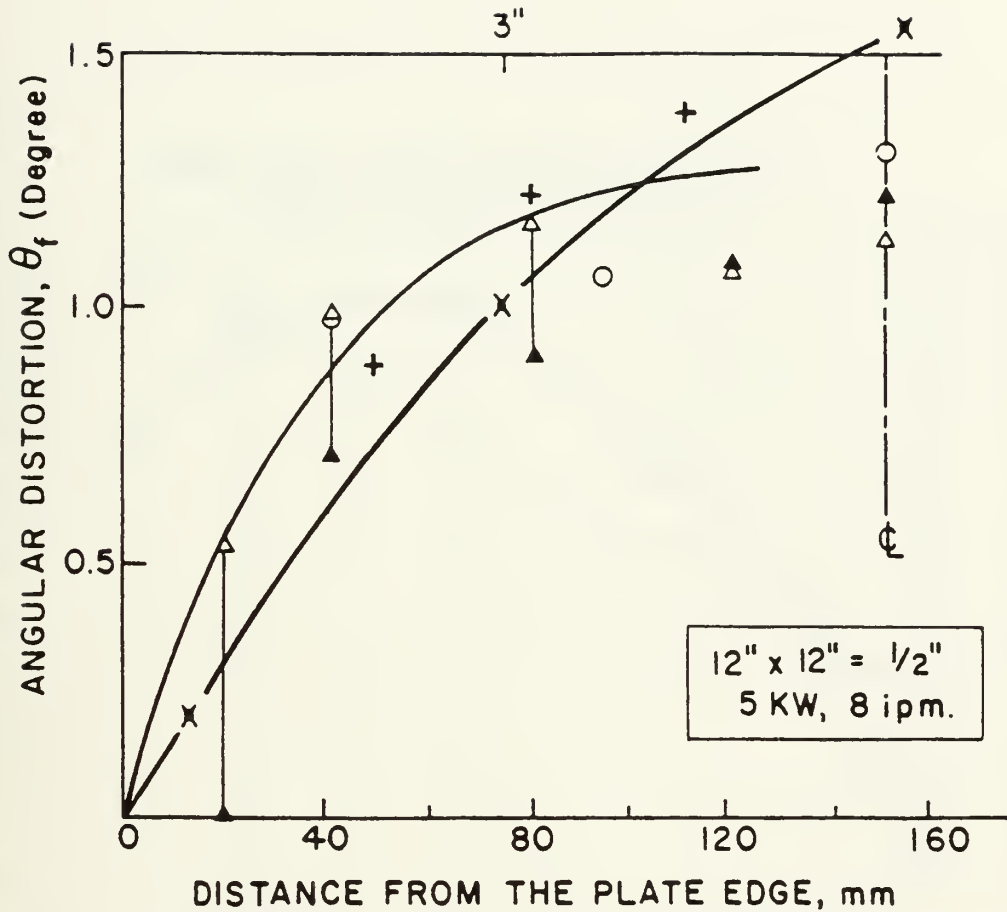
FIGURE 5.2   COMPARISON OF THE EFFECT OF LOCATION OF HEAT

SOURCE WITH THE EFFECT OF VARIATION IN PLATE LENGTH ON

ANGULAR DEFLECTION
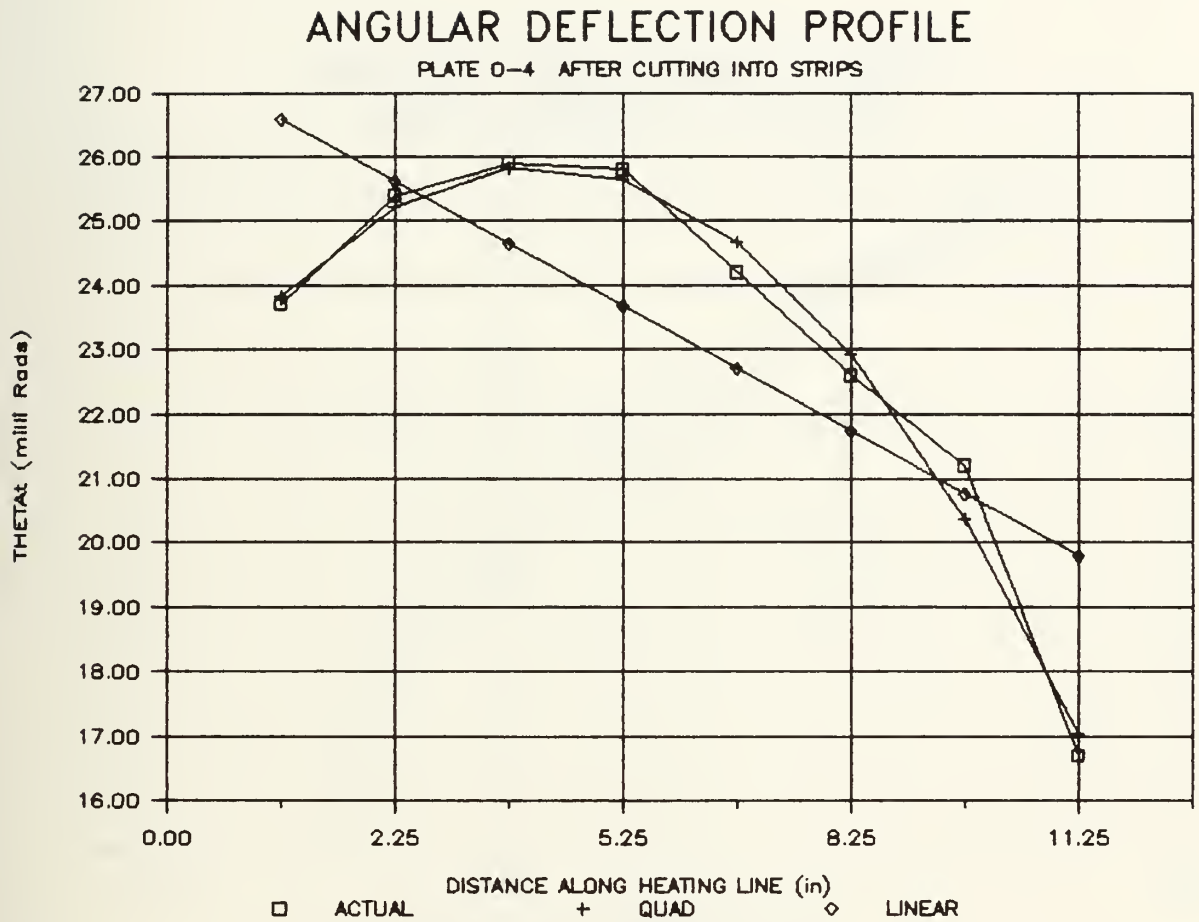
FIGURE 5.3   ANGULAR DEFLECTION PROFILE: PLATE 0-4 AFTER

CUTTING

FIGURE 5.4   ANGULAR DEFLECTION PROFILE: PLATE 1-3

ANGULAR DEFLECTION PROFILE

PLATE 1-4 DIMENSIONS: 12" X 18" X 3/8"


DISTANCE ALONG HEATING LINE (in)

□ ACTUAL + QUAD ◇ LINEAR
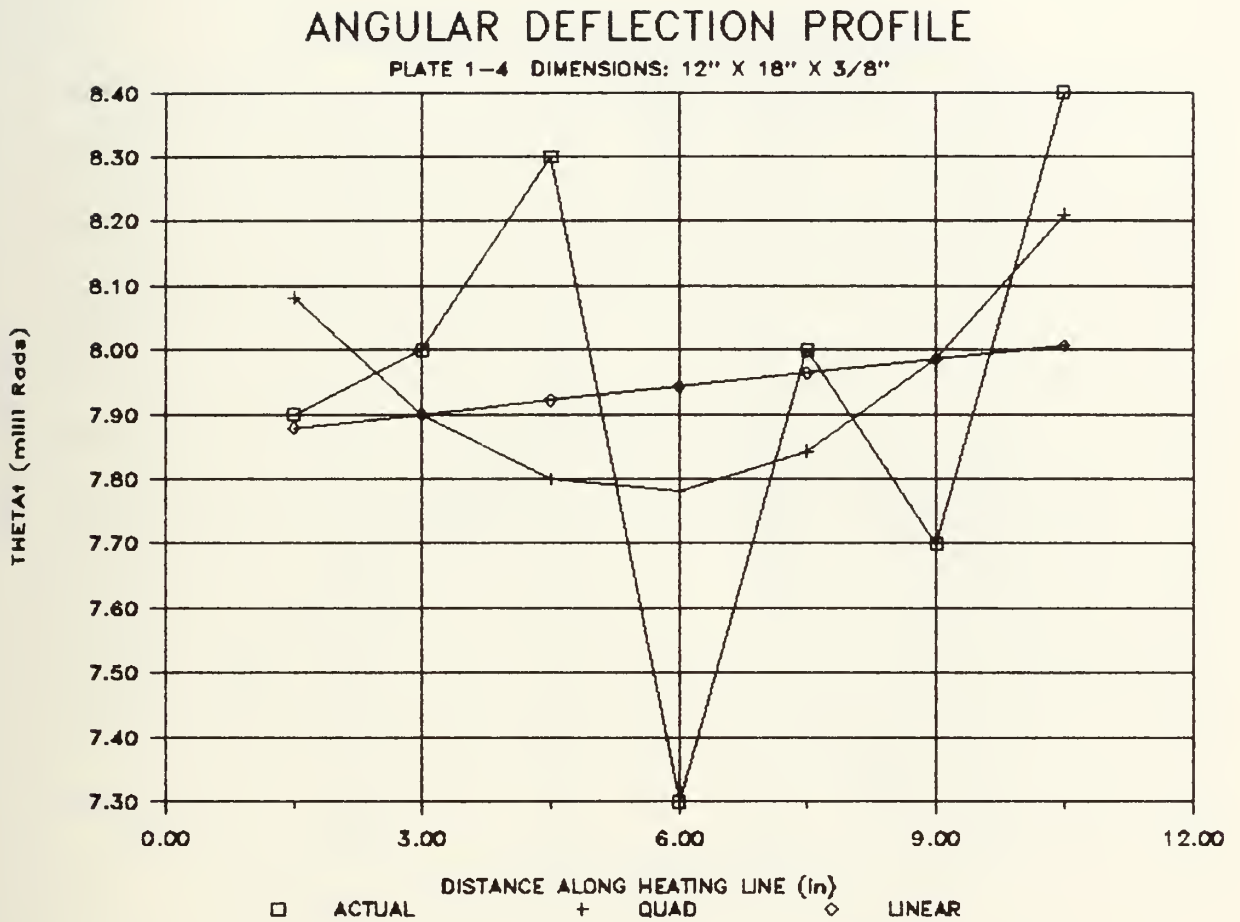
FIGURE 5.5 ANGULAR DEFLECTION PROFILE: PLATE 1-4

FIGURE 5.6 ANGULAR DEFLECTION PROFILE: PLATE 1-5

ANGULAR DEFLECTION PROFILE

PLATE 1-6 DIMENSIONS: 18" X 18" X 3/8"
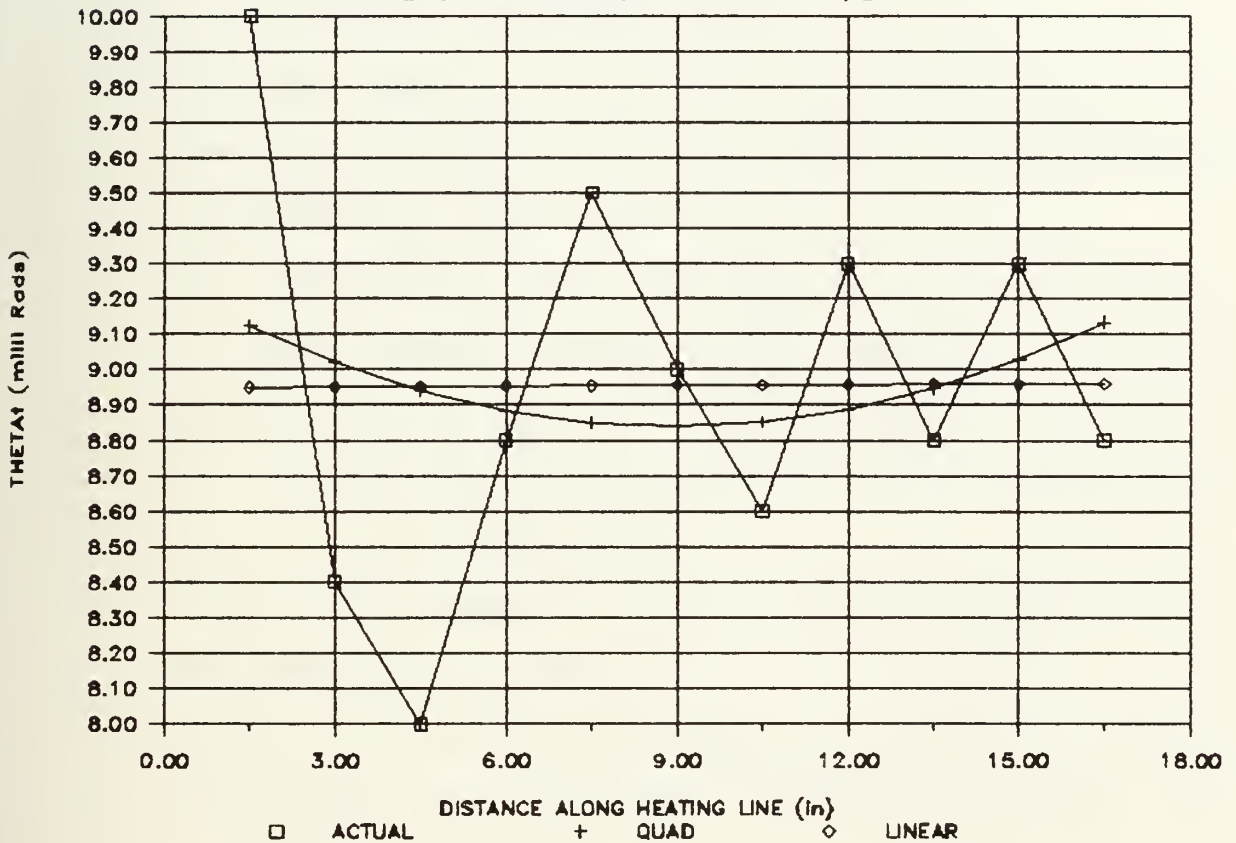
FIGURE 5.7 ANGULAR DEFLECTION PROFILE: PLATE 1-6

CHAPTER 6
CONCLUSIONS

The analysis presented leads to the following conclusions:

Angular deflection vs. plate length

1. The observed behavior of decreasing bending with decreasing length is a direct result of the increasing influence of "edge effects."

2. The average plate deflection for a given length, L, is found to be the same as that determined for heating parallel to an edge at a distance of L/2.

3. The behavior can be characterized by the relation:

$$\theta_t(L) = A(1 - e^{-BL}),$$

where the parameters A and B can be related to physical parameters as follows:

i) A is directly dependent on $P/V\bar{V}$ in the same manner that parametric studies have shown θ_t to depend on $P/V\bar{V}$.

ii) B is proportional to P/V^n with n estimated to be in the range of 0.5 to 1.5.

Angular deflection vs. in place location

1. The observed bending behavior along the line of heating varies in a manner similar to that of when the

distance between the line of heating to a parallel edge is varied, but is not symmetrical with respect to the plate center.

2.   This asymmetry is a consequence of the different orientation of the edge considered, and can be explained qualitatively from basic factors influencing bending, i.e. available material restraint.

3.   This variation in bending with location appears parabolic in nature subject to the following conditions:

i) the shape is concave down;

ii) the bending at the entrance edge is less than that at the exiting edge of the plate with respect to heat source movement.

4.   Of the two effects contributing to the bending at the edges (i.e. that of reduced material restraint and reduced temperature variation (gradient) through the plate), the reduced material restraint appears to dominate.

It should be noted that an important consequence of the above conclusions is that a model for predicting the actual bending at any point in a plate can be readily formulated.

The effects of plate length in the direction of heating and of the distance of the line of heating from a parallel edge on angular deflection can be determined as described above. Likewise, the effect of position along the line of heating on deflection could be determined using the parabolic characterization described above. The combination of these would provide the angular deflection at a given point. However, further studies of a parametric nature would be required to determine appropriate coefficients describing the parabolic behavior.

In summary, the conclusions presented above provide a better understanding of the bending behavior in plates of varying dimensions (length and width). Additionally, this behavior can be characterized in terms of parameters related to physical inputs to the thermo-mechanical forming process i.e. power and speed. Thus permitting the development of a model capable of predicting bending that can incorporate the actual dimensions of the plate, and the location and orientation of the proposed line of heating. This predictor model can be readily integrated with existing algorithms used to predict the general large plate deflection. Moreover, this model could be extended to include actual variations in bending within the plate if additional studies can verify and characterize the apparent parabolic behavior

observed.   Therefore, the conclusions achieved in this study
have contributed significantly to answering the questions
that originally motivated the investigation.

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(Continued)

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   Stresses." John Wiley & Sons, Inc., 1960

APPENDIX

PARAMETRIC STUDY

The graphical results of fitting a curve of the form,

$$\theta_t(L) = A(1 - e^{-BL}),$$

to the plate length vs. bending data for flame and laser heating are contained in this appendix. The parametric values for each case studied have been summarized in **Table 5.1.**
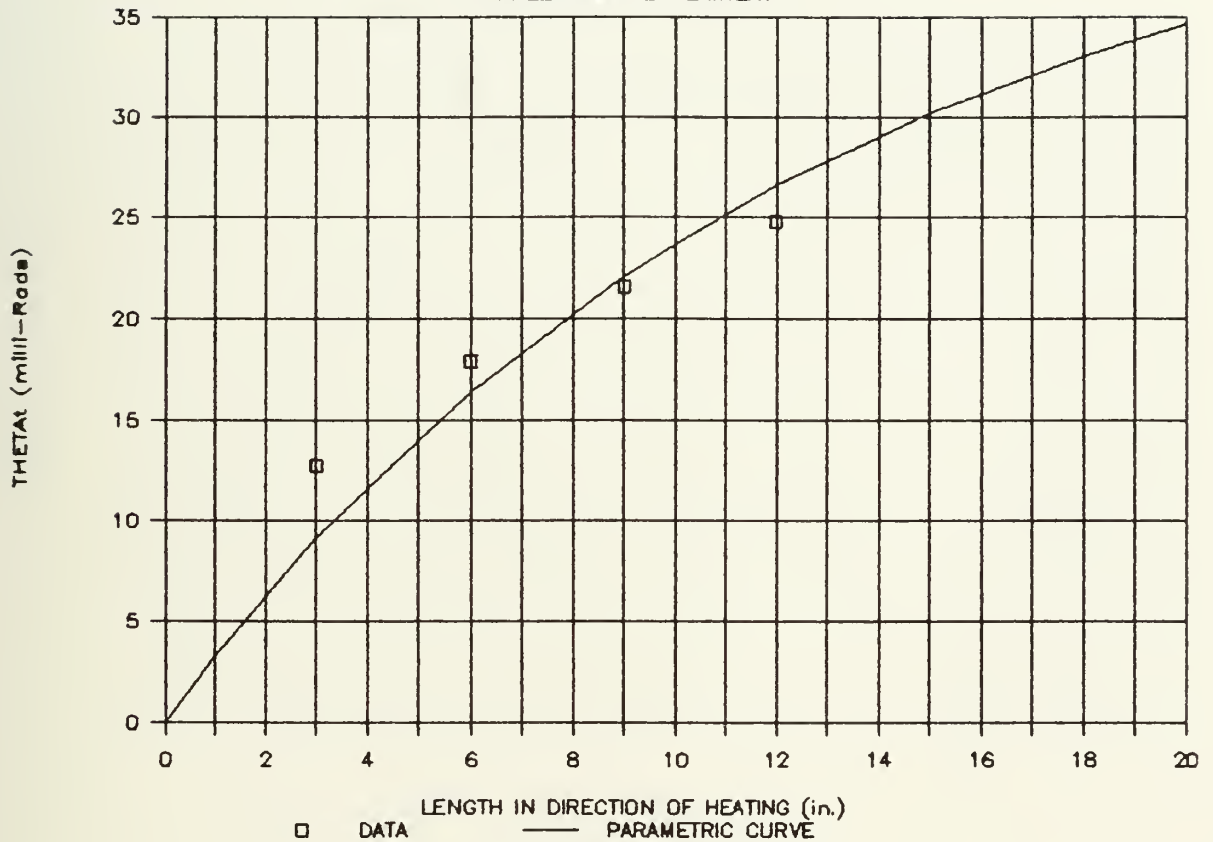
FIGURE A.1   FLAME HEATING 3/8" PLATE-- PRELIMINARY
EXPERIMENT

PARAMETRIC STUDY

FOLLOW-ON EXPERIMENT # 1
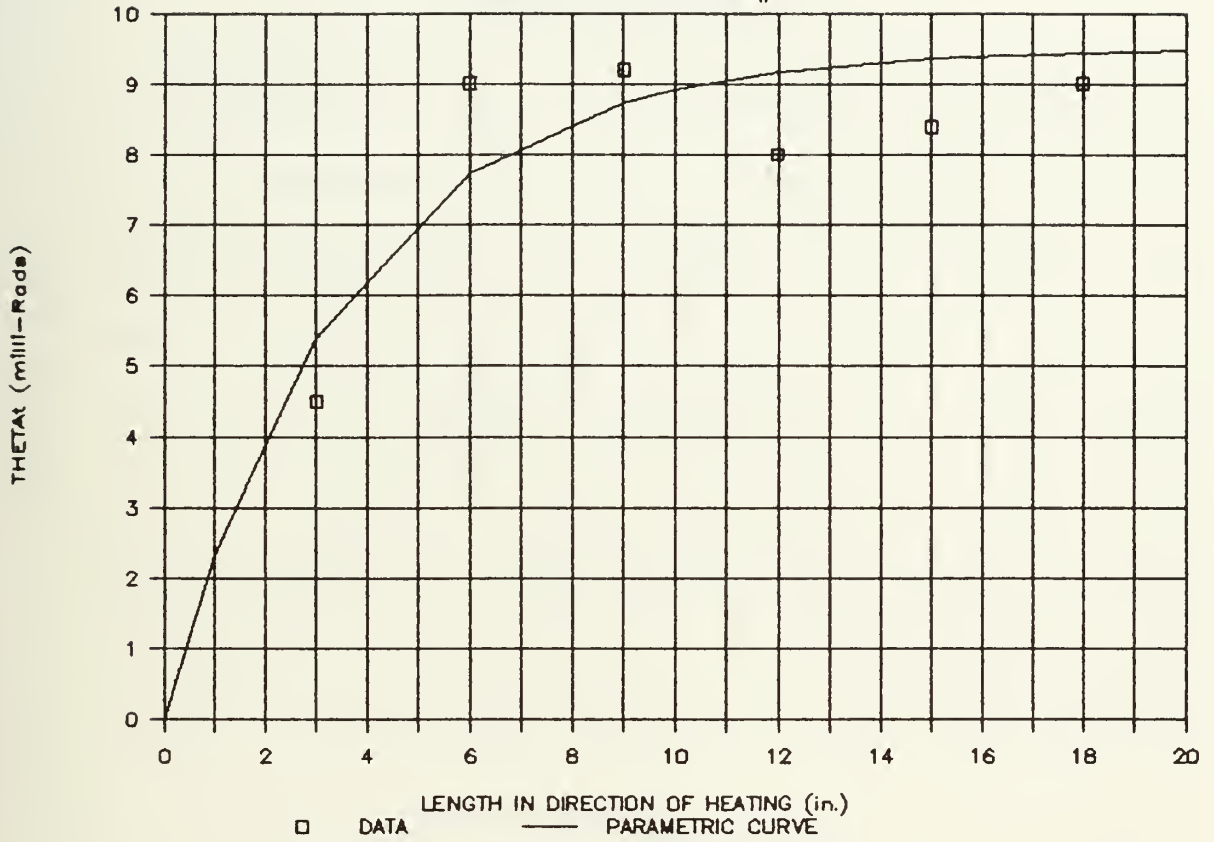
A=9.50 B=0.280

FIGURE A.2 FLAME HEATING 3/8" PLATE-- FIRST FOLLOW-ON
EXPERIMENT

PARAMETRIC STUDY

FOLLOW—ON EXPERIMENT # 2
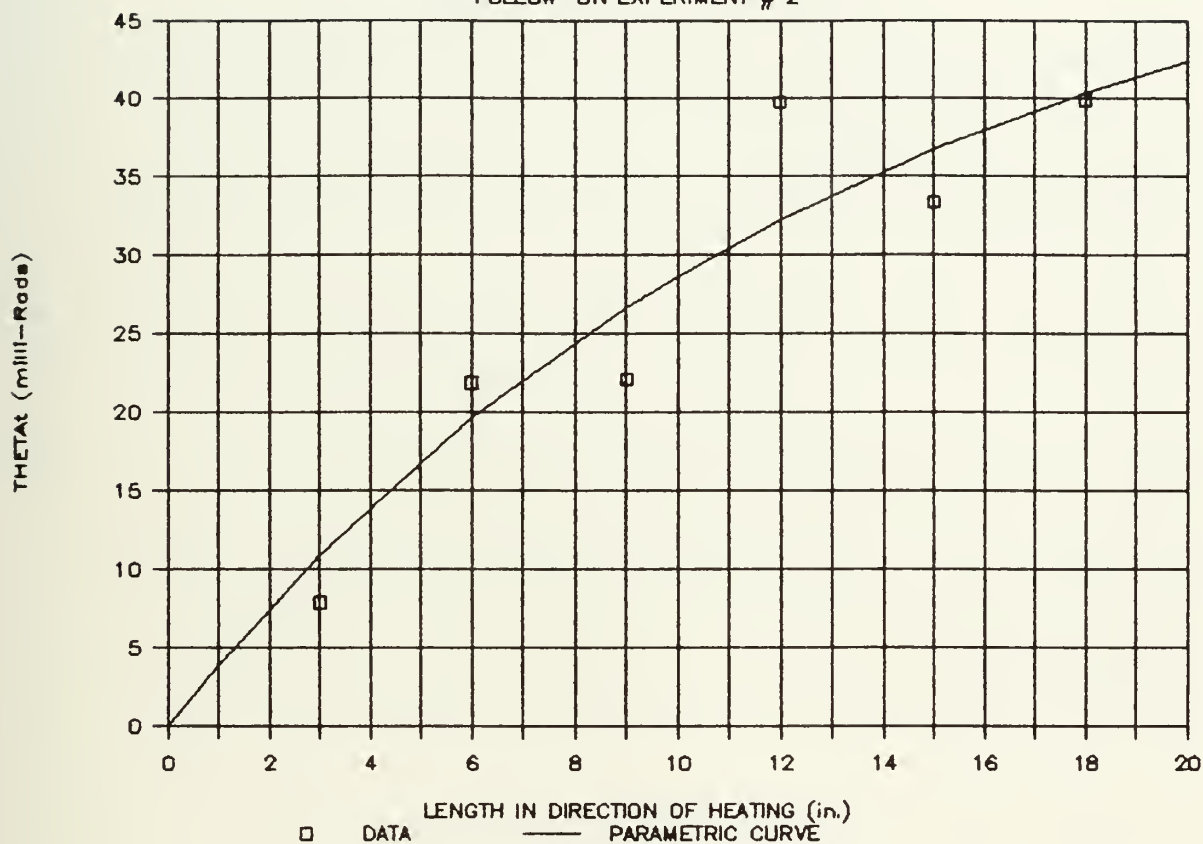
A=55.00 B=0.074

FIGURE A.3 FLAME HEATING 3/8" PLATE-- SECOND FOLLOW-ON
EXPERIMENT

PARAMETRIC STUDY

AVG. OF PRELIM. & FOLLOW-ON EXP. # 2
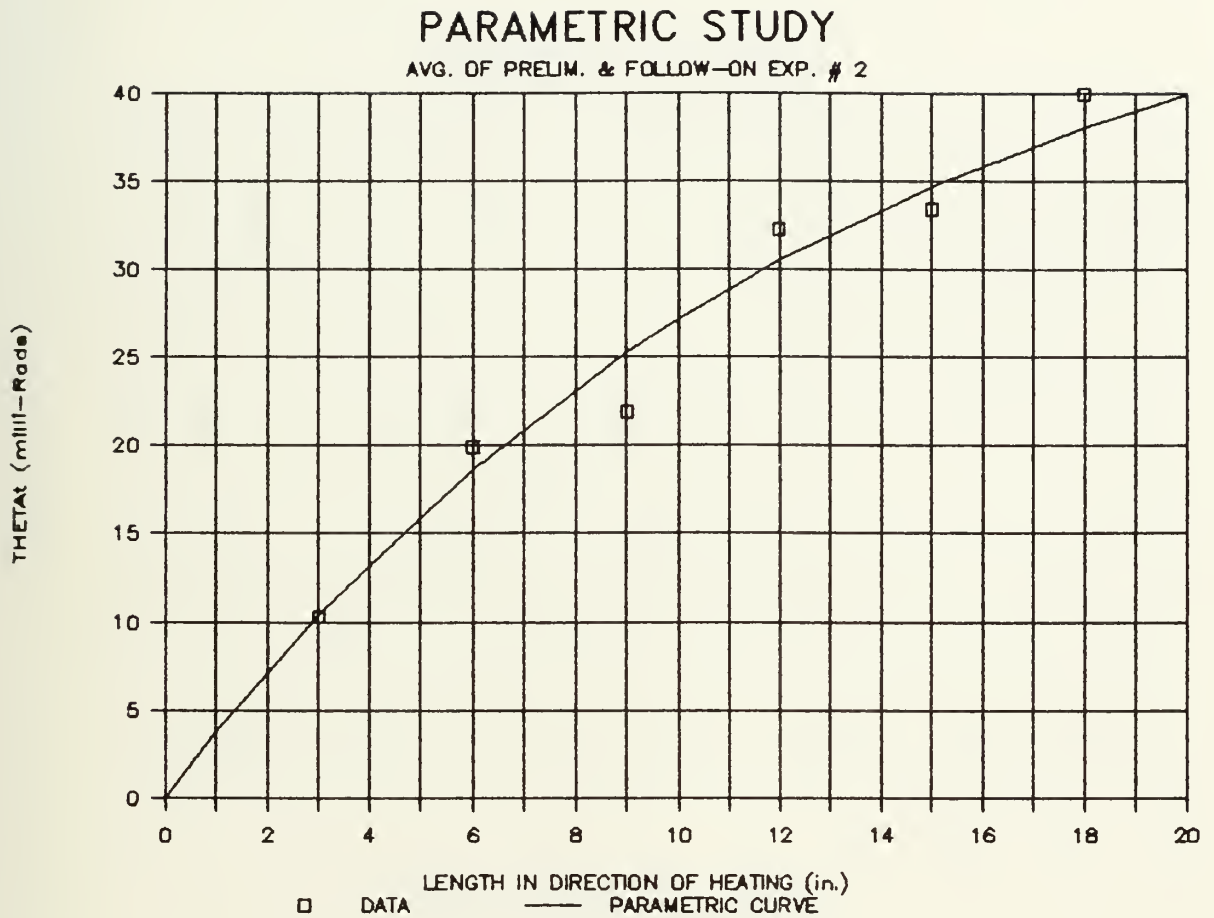
A=51.70 B=0.075

FIGURE A.4 FLAME HEATING 3/8" PLATE-- AVERAGE OF
PRELIMINARY & SECOND FOLLOW-ON EXPERIMENT

PARAMETRIC STUDY

LASER DATA: P/√v̄ = 2.0, t=3/8
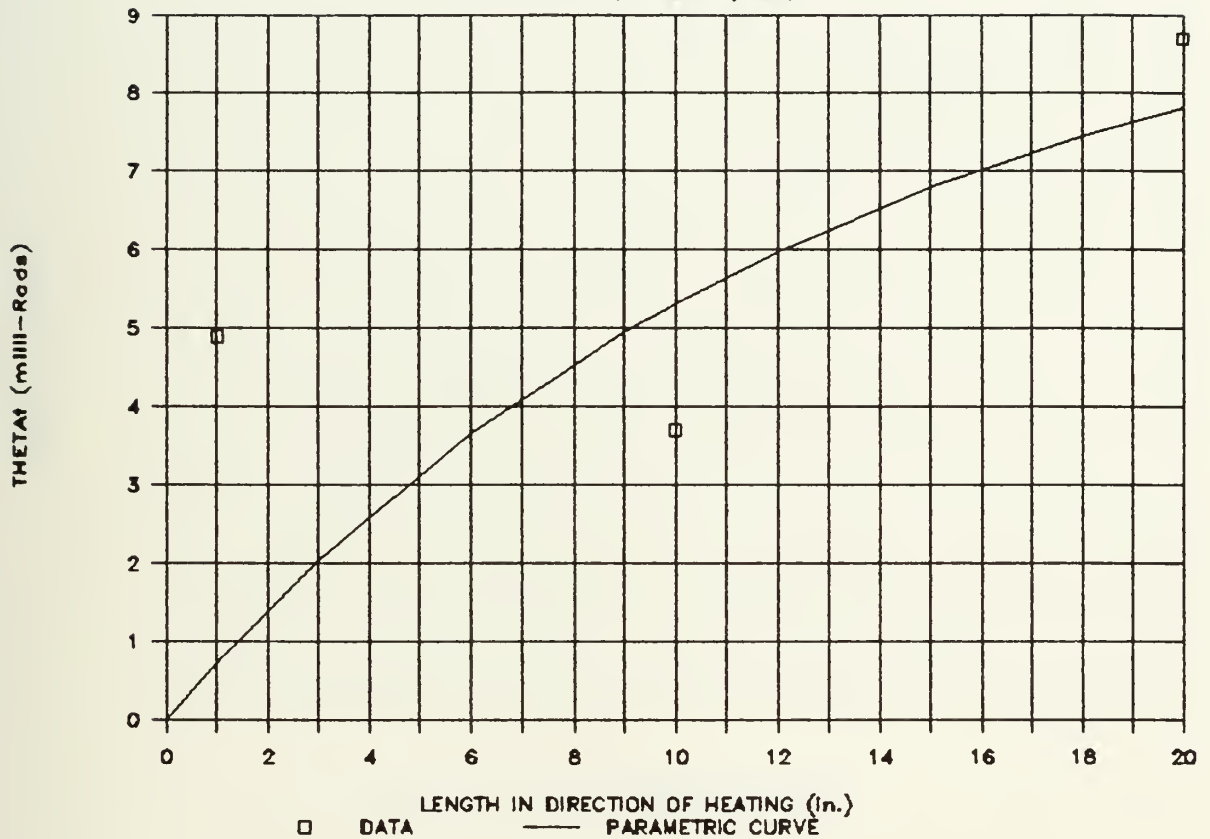
A=10.00 B=0.076

FIGURE A.5   LASER HEATING 3/8" PLATE-- # 1

PARAMETRIC STUDY

LASER DATA: P/W = 2.5, t-3/8
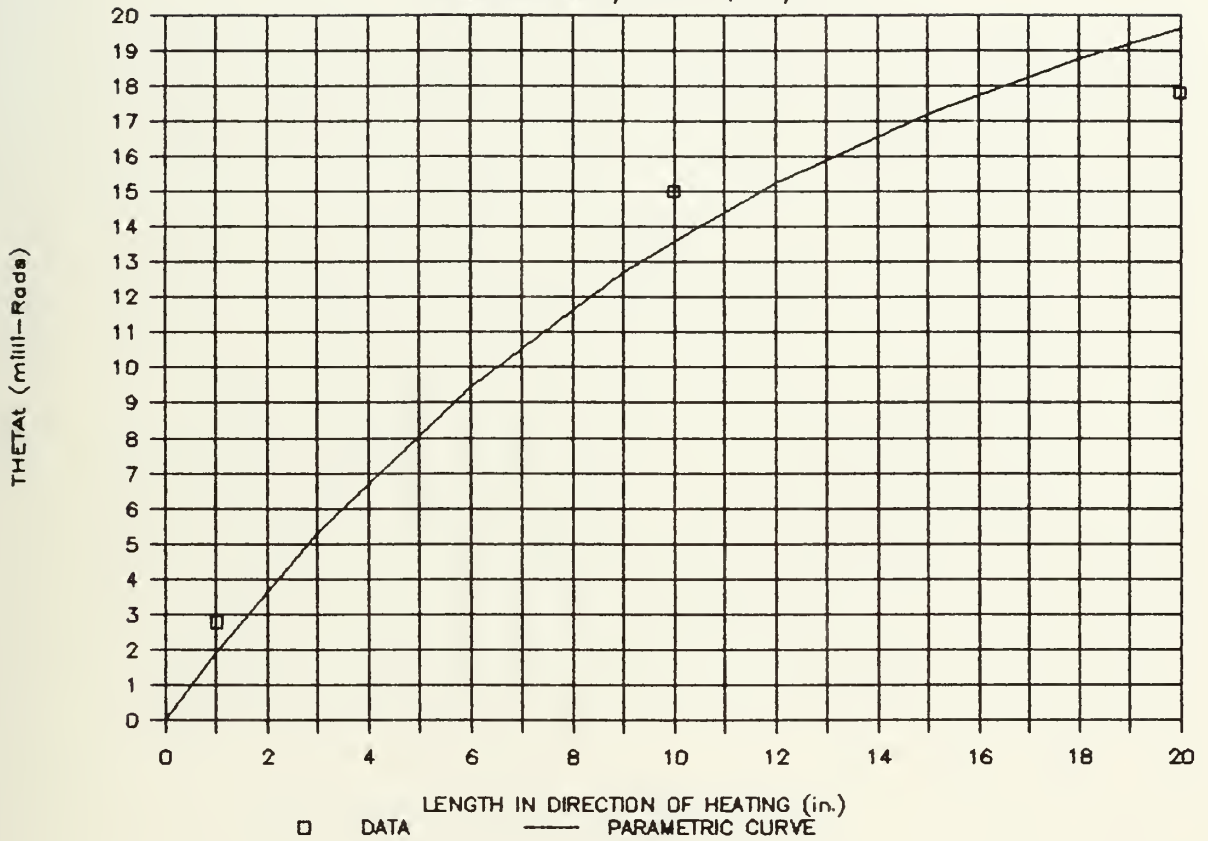
A=24.30 B=0.082

FIGURE A.6   LASER HEATING 3/8" PLATE-- # 2

PARAMETRIC STUDY

LASER DATA: AVG. P/√v̄ = 2.3, t=3/8
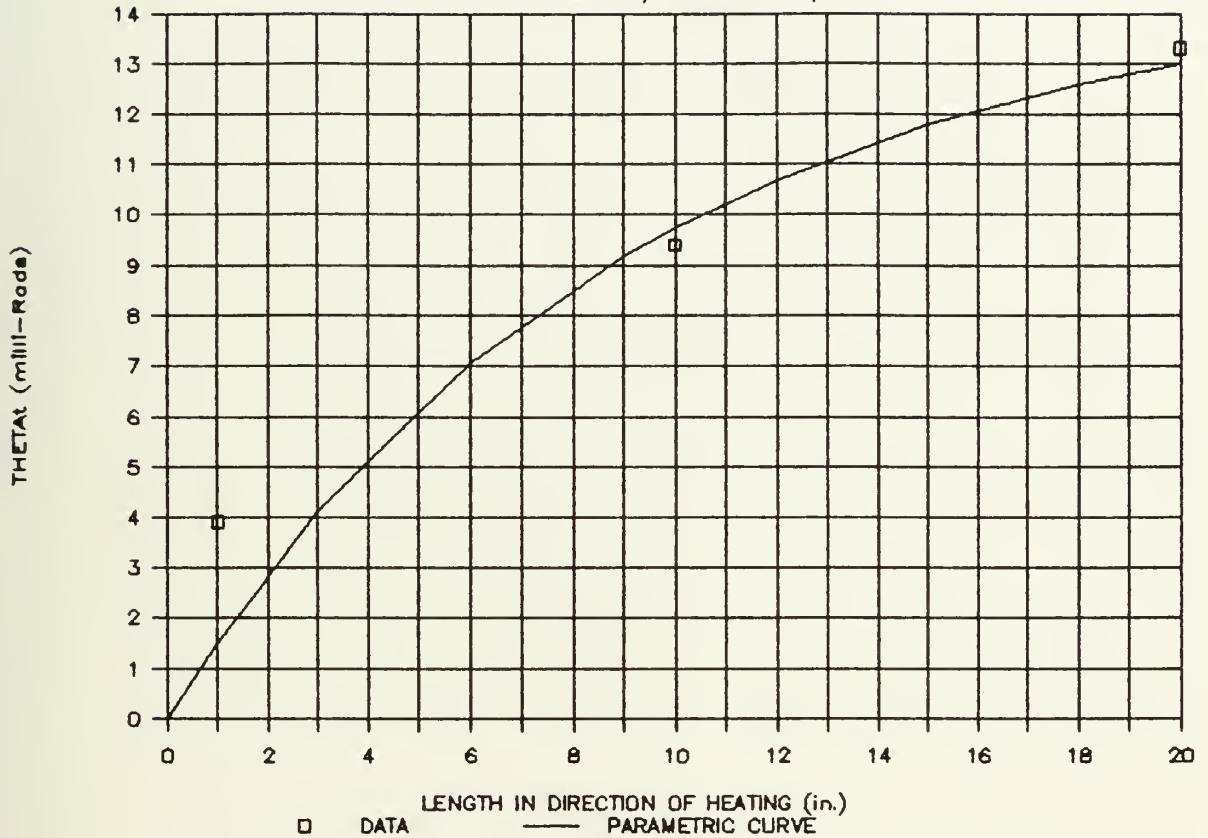
A=14.60 B=0.110

FIGURE A.7   LASER HEATING 3/8" PLATE-- AVERAGE OF # 1 & # 2

PARAMETRIC STUDY

LASER DATA: P/W — 1.8, t—1/2
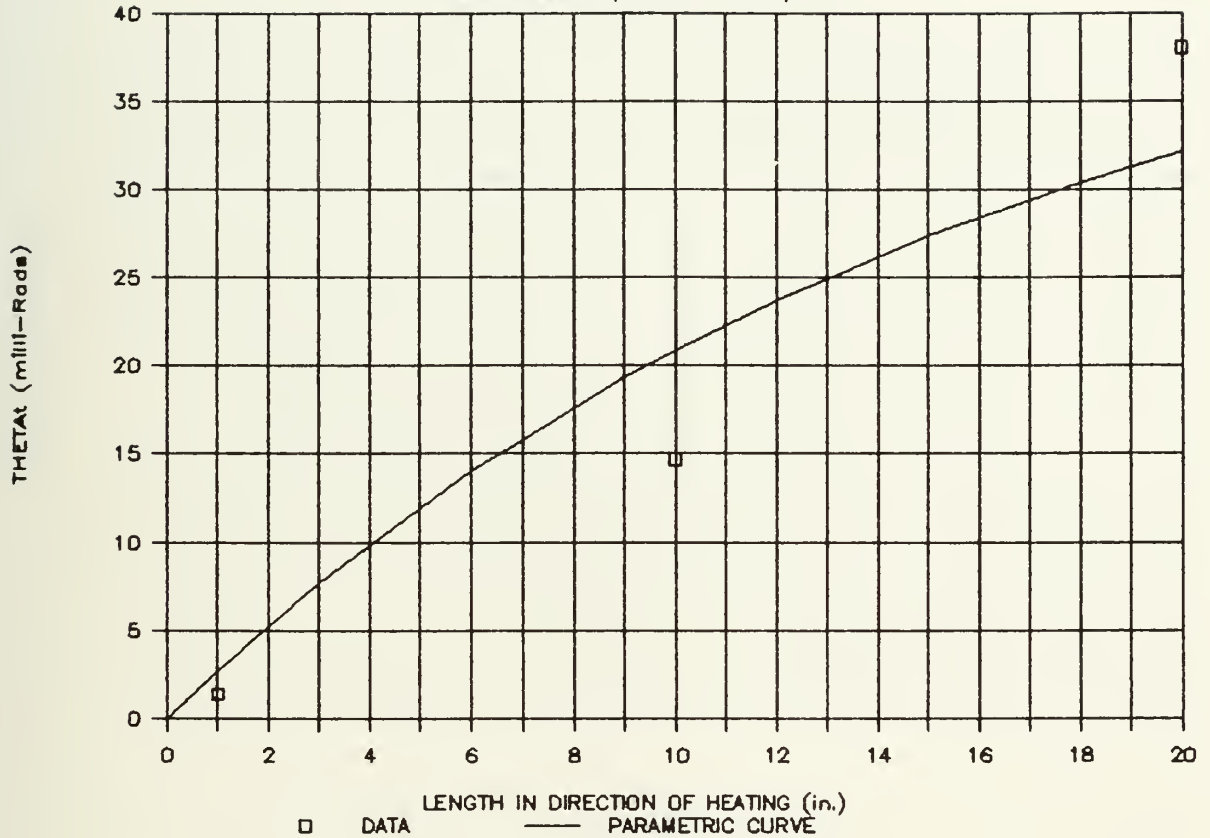
A=45.20 B=0.062

FIGURE A.8   LASER HEATING 1/2" PLATE-- # 3

PARAMETRIC STUDY

LASER DATA: P/√W — 2.5, t—1/2
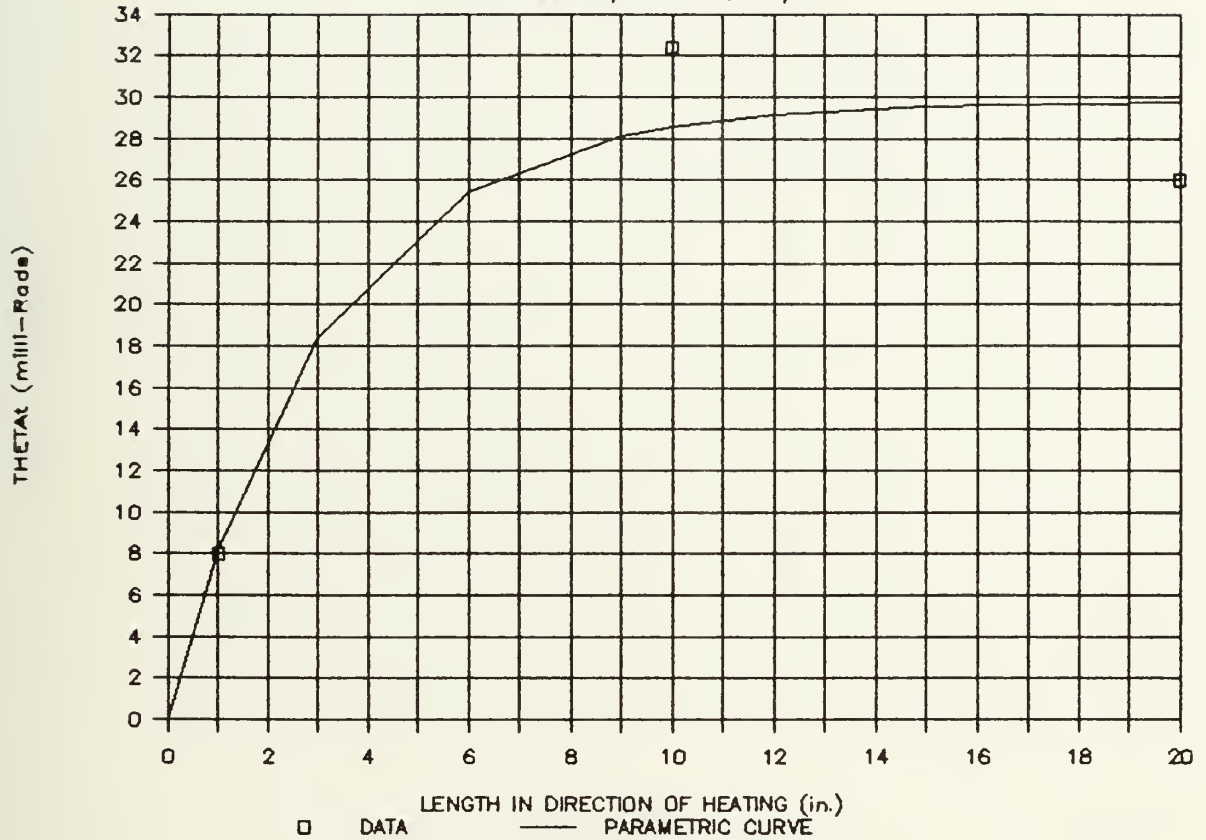
A=29.80 B=0.320

FIGURE A.9   LASER HEATING 1/2" PLATE-- # 4

PARAMETRIC STUDY

LASER DATA: AVG. P/$\sqrt{v}$ — 2.1, t—1/2
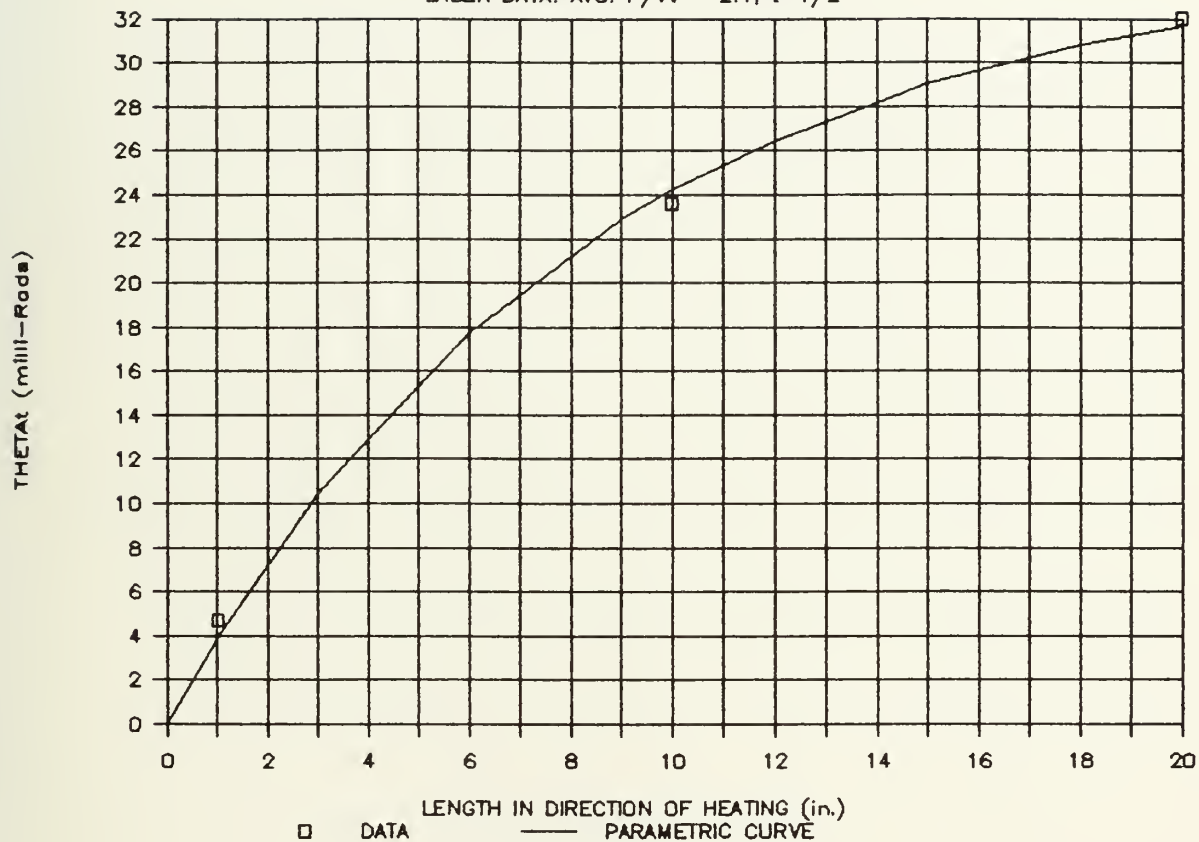
A=35.00 B=0.118

FIGURE A.10   LASER HEATING 1/2" PLATE-- AVERAGE OF # 3 & # 4

PARAMETRIC STUDY
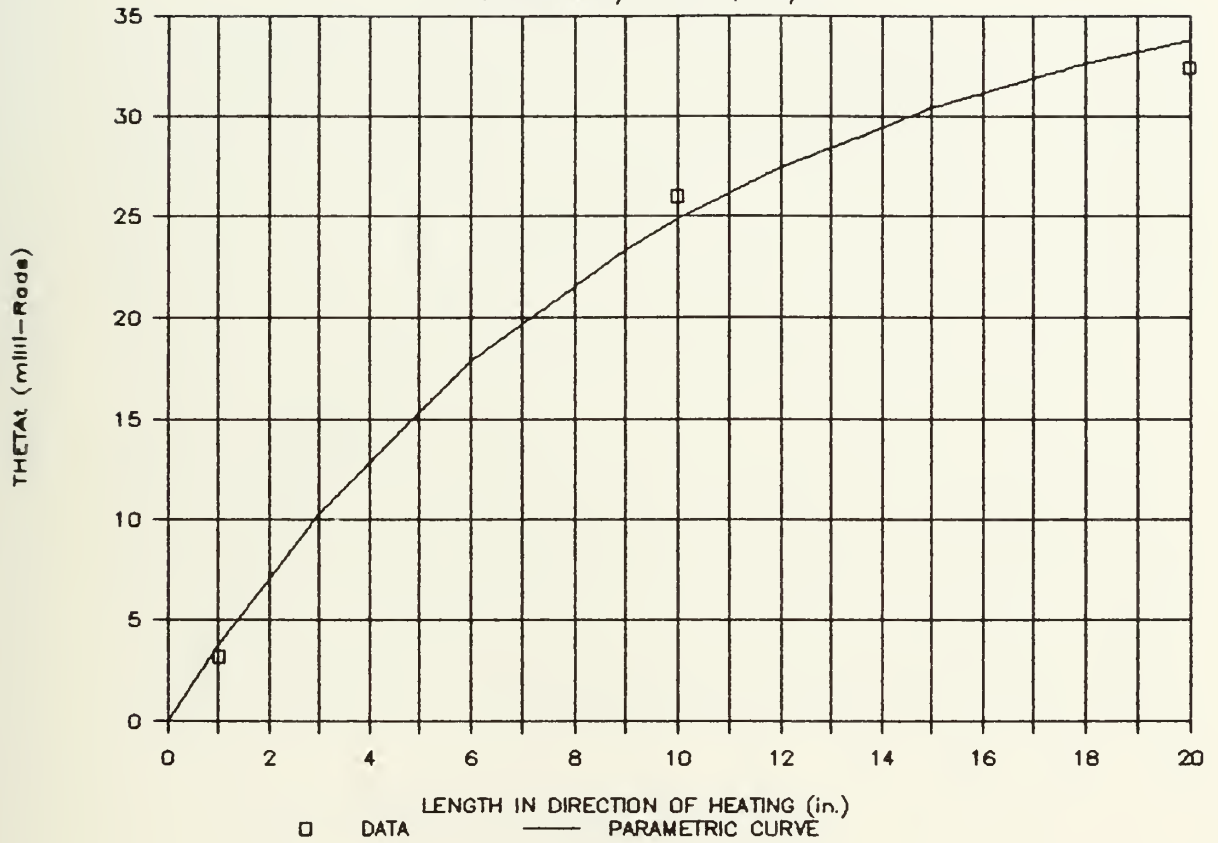
LASER DATA: P/√W̄ — 3.0, t—5/8

FIGURE A.11   LASER HEATING 5/8" PLATE-- # 5

A=38.70 B=0.103

PARAMETRIC STUDY

LASER DATA: P/√W̅ — 3.5, t=5/8
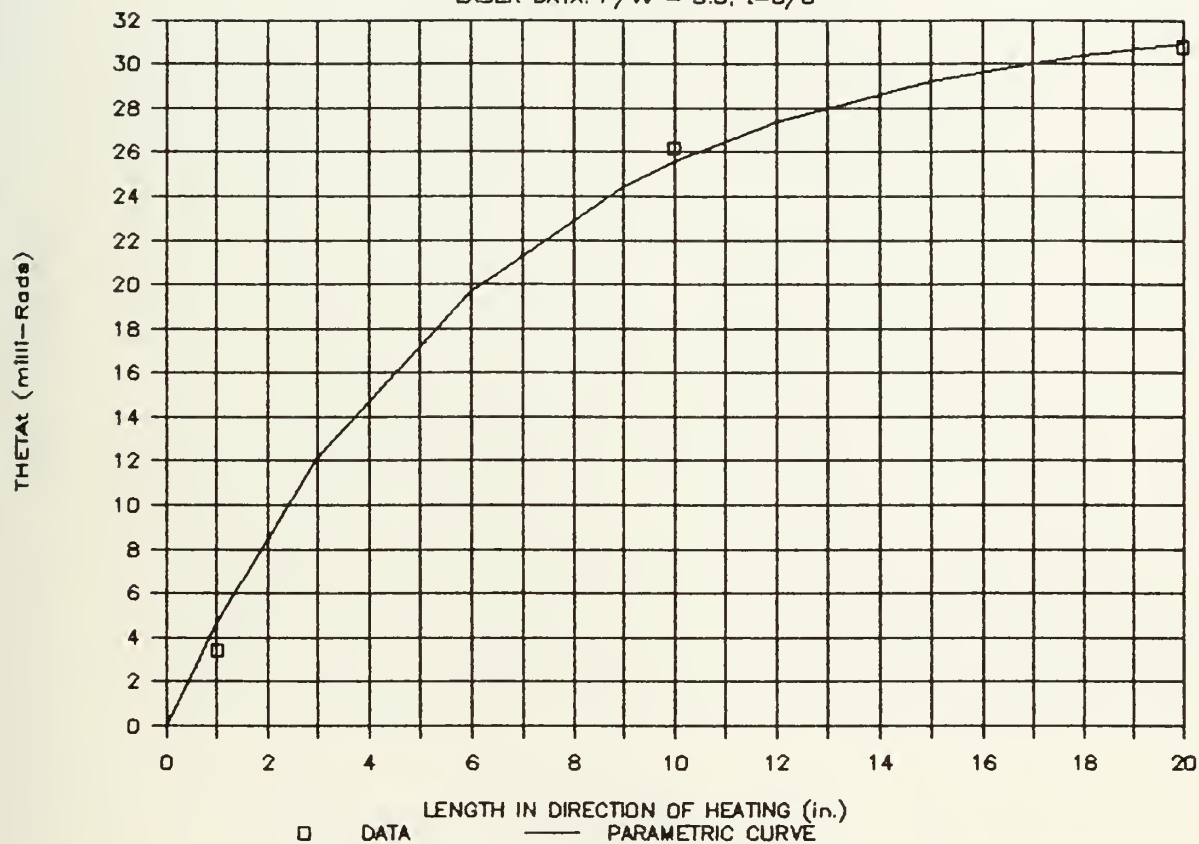
A=32.30 B=0.157

FIGURE A.12   LASER HEATING 5/8" PLATE-- # 6

PARAMETRIC STUDY

LASER DATA: AVG. $P/\sqrt{v}$ = 3.3, t=6/8
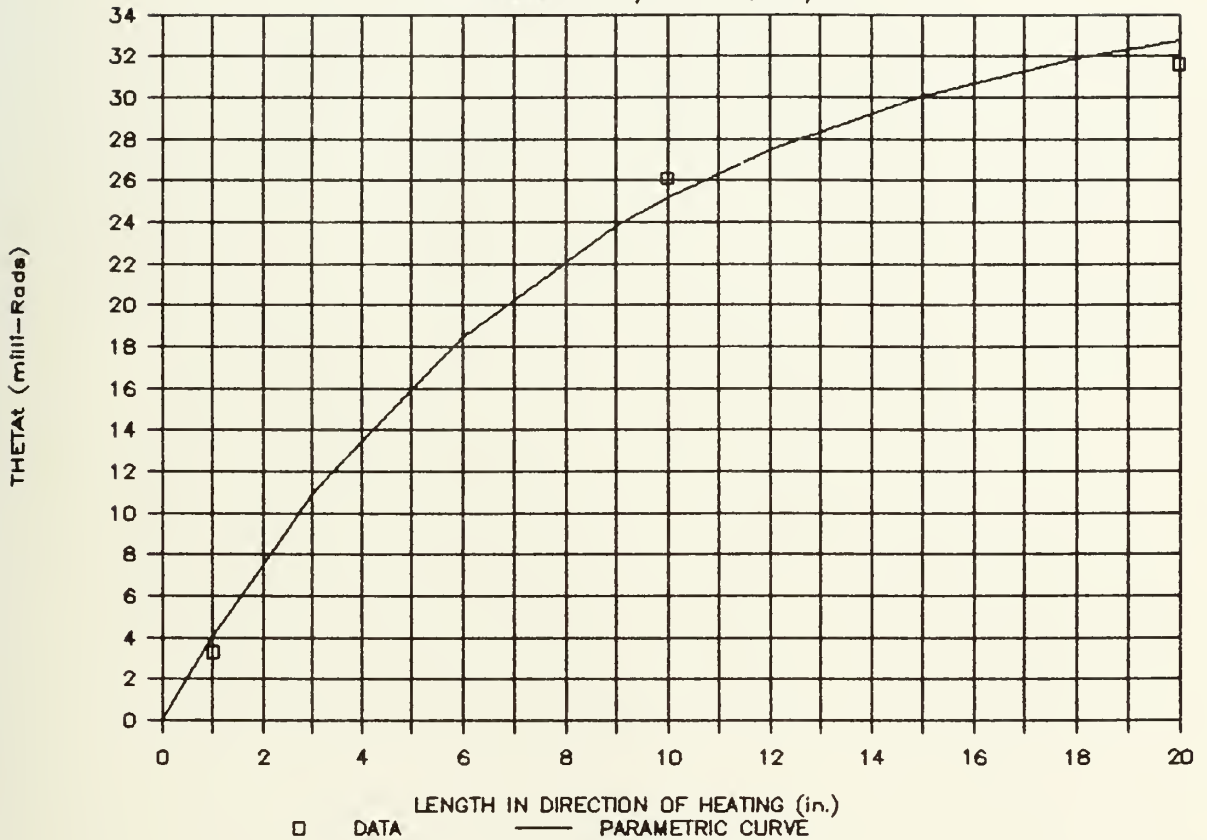
A=36.03 B=0.121

FIGURE A.13   LASER HEATING 5/8" PLATE-- AVERAGE OF # 5 & # 6